Endogenous R&D and Technology Diffusion in a Multi-Sector RBC Economy

Masters Dissertation by Sebastian Krantz

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Abstract

Starting from J. B. Long & Plosser (1983), multi-sector DSGE models have challenged the notion of aggregate technology shocks as the main driver of business cycle fluctuations, and provided a strong evidential basis for the relevance of independent sectoral shocks amplified by asymmetric production networks. More recently, with Comin & Gertler (2006) short-run conceptions of the business cycle were challenged, and medium-run cycles are modelled in terms of endogenous technology mechanisms. This dissertation presents a pioneering attempt to reach a synthesis between these different strands of literature. It introduces a multi-sector RBC economy in which each sector independently makes decisions to invest in R&D and adopt new technologies. Sectors interact with each other in a productive network via intermediate inputs, but are also affected by R&D and technology adoption spillovers transmitted from upstream or downstream sectors in the value chain. Evidence is provided that this model is capable of accounting for extended observed aggregate and sectoral fluctuations.

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1 Introduction

This master thesis proposes, builds and simulates of a new form of macroeconomic Dynamic Stochastic General Equilibrium (DSGE) model which provides a rich and disaggregated account of production in a multi-sector Real Business Cycle (RBC) economy. In contrast to other multi-sector models describing input-output interactions in the economy such as J. B. Long & Plosser (1983), Horvath (2000) or Atalay (2017), sectors in this model comprise not only of monopolistically competitive firms employing labor, capital and intermediate inputs in production, but also of a set of technology adopters and technology innovators. Adopters and innovators each employ skilled labor to adopt new technologies (i.e. convert an idea or technology into a production plan) and invent new technologies, respectively. With a fully decentralized production and innovation system in place in each sector, sectoral interactions in this model occur in terms of value chains in intermediate inputs, but also in the form of sectoral spillovers in technology adoption and R&D.

The intentions behind this model are several. First, to provide a richer account of modern production processes and study technology diffusion and R&D spillovers through the input-output network. Secondly a model that is able to describe short-and medium-term fluctuations in aggregate and sectoral outputs and productivities without excessive reliance on nominal frictions[1] or exogenous technology, as is the predominant approach in the New-Keynesian DSGE literature. Thirdly, the model cuts a bridge between two active but rather distinct literatures in macroeconomics: the literature on sectoral shocks and aggregate fluctuations with key contributions by J. B. Long & Plosser (1983), Horvath (1998, 2000), Petrella & Santoro (2011), Acemoglu et al. (2012, 2016), Bouakez et al. (2014), Stella (2015), and Atalay (2017) amongst others, and the literature on medium-run fluctuations and the integration of growth and business cycles with key contributions by Comin & Gertler (2006), Comin (2009), Bianchi et al. (2018) and Anzoategui et al. (2017). The former literature attempts to model the multi-sector economy in intermediate productive inputs and has focussed on the question to what degree independent and uncorrelated sectoral shocks can generate, or are responsible for, aggregate business cycle volatility. Most of the models constructed in this literature are simple Real Business Cycle (RBC) models that are carefully calibrated for 20-40 sectors in the US economy (2-digit Standard Industrial Code (ISIC) level). The results from this literature show a remarkable success in generating aggregate US volatility from such a set-up without any nominal frictions - in stark contrast to the dominant New-Keynesian approach that has relied on many such frictions to generate volatility in aggregate data. The multi-sector literature has also determined under which conditions independent and uncorrelated sectoral shocks can generate aggregate volatility. The second literature focuses on modelling and explaining the "medium-run cycle" - a concept first introduced by Comin & Gertler (2006) to denote the combination of high and medium-frequency components of (US) output and productivity that fluctuate around a very smooth non-linear trend at frequencies of up to 200 quarters. The degree of aggregate volatility studied is thus much greater and much longer than that captured in the traditional business cycle, defined by the frequencies up to 32 quarters. The claim of Djego Comin and his co-authors, established through their models and supporting empirical evidence, is that this medium-run cycle constitutes neither fluctuations in long-run growth nor a completely separate phenomenon. Rather it interacts with, and is driven by, the classical business cycle. According to Comin this interaction takes place in the form of pro-cyclical decision by firms and other economic agents to invest in R&D and adopt new technologies, which let the business cycle have medium-run effects. Strategic decisions become the driving force behind the medium-run cycle. The models developed by these authors to explain the medium-run cycle, culminating in Anzoategui et al. (2017), are essentially New-Keynesian business cycle models with many frictions which include an extended endogenous R&D and technology adoption mechanism.

Considering the results of these two literatures in macroeconomics, it is evident what the contribution of the model I develop in this dissertation is: If the business cycle can be explained to a significant degree by sectoral interactions, and if business cycle decisions to invest in R&D and technology adoption translate into medium-run fluctuations in output and productivity that far outlive the business cycle, then these decisions should also be explicable to a good extent in a framework of sectoral interactions. In other words, sectoral interactions might be a major driv-

[1] Sectoral frictions in prices, wages and investment adjustment costs may still be added to provide additional realism, but are not implemented in the baseline model exhibited in this dissertation.

ing force behind both business and medium-run cycles. To investigate this possibility I create a multi-sector RBC model that includes imperfect competition and a pro-cyclical and fully endogenous production and innovation system in each sector - modelled after Anzoategui et al. (2017). I then introduce several mechanisms by which sectoral interactions could give rise to aggregate medium-run fluctuations. The first and most intuitive of these mechanisms operates by linking all productive sectors through intermediate input (and demand) linkages, thereby allowing independent sectoral shocks to transmit through the input-output network while generating pro-cyclical responses in terms of R&D and technology adoption decisions in different sectors. The second mechanism works via strategic complementarities in technology adoption inside productive value chains: If downstream sectors adopt a new technology that allows them to produce greater volumes more efficiently and more profitable, they will require greater volumes of intermediate inputs from upstream suppliers. This will induce the upstream sectors to also increase their productive efficiency by adopting new technologies. Likewise upstream sectors could adopt a new technology enabling them to supply greater volumes of intermediate inputs more cheaply. With some competitive pressure in downstream sectors, this will depress the price of downstream products and increase demand - requiring also downstream sectors to adopt new technologies and produce greater volumes. A third possibility, albeit a theoretically more ambiguous one, are spillovers in the productivity of R&D across different sectors. It is conceivable for example that a technological breakthrough in optics or nano-technology leads, with little delay, to a similar breakthrough in consumer-electronics - a more rapid phase of new products being invented and making it to the market. Likewise a breakthrough in consumer electronics - for example in electric cars, could increase R&D in some upstream sectors i.e. firms working on batteries. In allowing for these adoption and R&D spillovers, my aim is also to impose as little restrictions as possible on the kind of sectoral interactions allowed to take place - preferably letting the data speak in an estimation exercise.

These things being said, I have unfortunately to this point not been able to fulfil the aim of my research: in the following I will construct, solve and succesfully simulate a model of the kind just outlined, but due to limitations in the amount of time and the quality of data available to me, I have not yet been able to evaluate the model or study its precise properties in an estimation exercise. This failure to truly evaluate the model is largely due to the long time needed to construct the model itself and problems encountered thereby, some of which still demand a more satisfactory solution than presently implemented. Below I therefore spend some time laying out in detail the construction of the model in a way that does justice to its complexity. I will do this in two steps: First, I construct a multi-sector RBC model and study its properties in a simulation. Then I will present a simple RBC model of endogenous technology, where I will implement the full endogenous R&D and technology diffusion mechanism devised by Anzoategui et al. (2017), but in a much simpler model than Anzoategui et al. (2017), and also study its properties by means of a simulation. Having thus studied the two key ingredients - sectoral interaction in intermediate inputs and endogenous R&D and technology adoption in a single sector - I will construct the full model by integrating these two models into one-another. The properties of the full model will also be studied by means of simulations. Finally I calibrate both the multi-sector RBC and the full model to the 3-sector US economy in agriculture, industry and services, followed by an extended calibration of the multi-sector RBC to the 10-sector US economy. The calibration results are then compared with the results from a Structural VAR (SVAR). The purpose of this final calibration exercise is to establish that the model is, at least in theory, capable of fitting the data - a task that is to be taken further in a careful estimation of the model by Bayesian methods.

The remainder of this document is structured as follows: section 2 very briefly reviews the literatures on sectoral shocks and aggregate fluctuations and on medium-run fluctuations and endogenous-technology DSGE models. Section 3 introduces and simulates the basic multi-sector RBC model and section 4 introduces and simulates the basic RBC model with endogenous R&D and technology diffusion. Section 5 then integrates the two models to yield the model advocated above and provides simulation results for a basic 2-sector set-up. Section 6 contains the calibration exercise to the 3- and 10-sector US economy. Section 7 concludes.

2 Literature Review

The literature on multi-sector DSGE models is long and dates back principally to J. B. Long & Plosser (1983). While multi-sector and multi-region DSGE models are becoming more frequent in recent macroeconomic analysis as a means to allow for sectoral interactions and heterogenous effects of policy, not all papers address, implicitly or explicitly, the objective set by J. B. Long & Plosser (1983) and following papers, which is to study how aggregate fluctuations can be the result of sectoral shocks in an economy where productive sectors are linked through input-output interactions. It is this more-limited multi-sector DSGE literature that I will briefly review in the following subsection. Likewise many models feature some form of endogenous technology mechanism, but only a handful of papers study how decisions to invent and adopt new technologies, informed by economic considerations of decentralized agents, translate into aggregate R&D and technology diffusion and dynamics in aggregate economic variables.

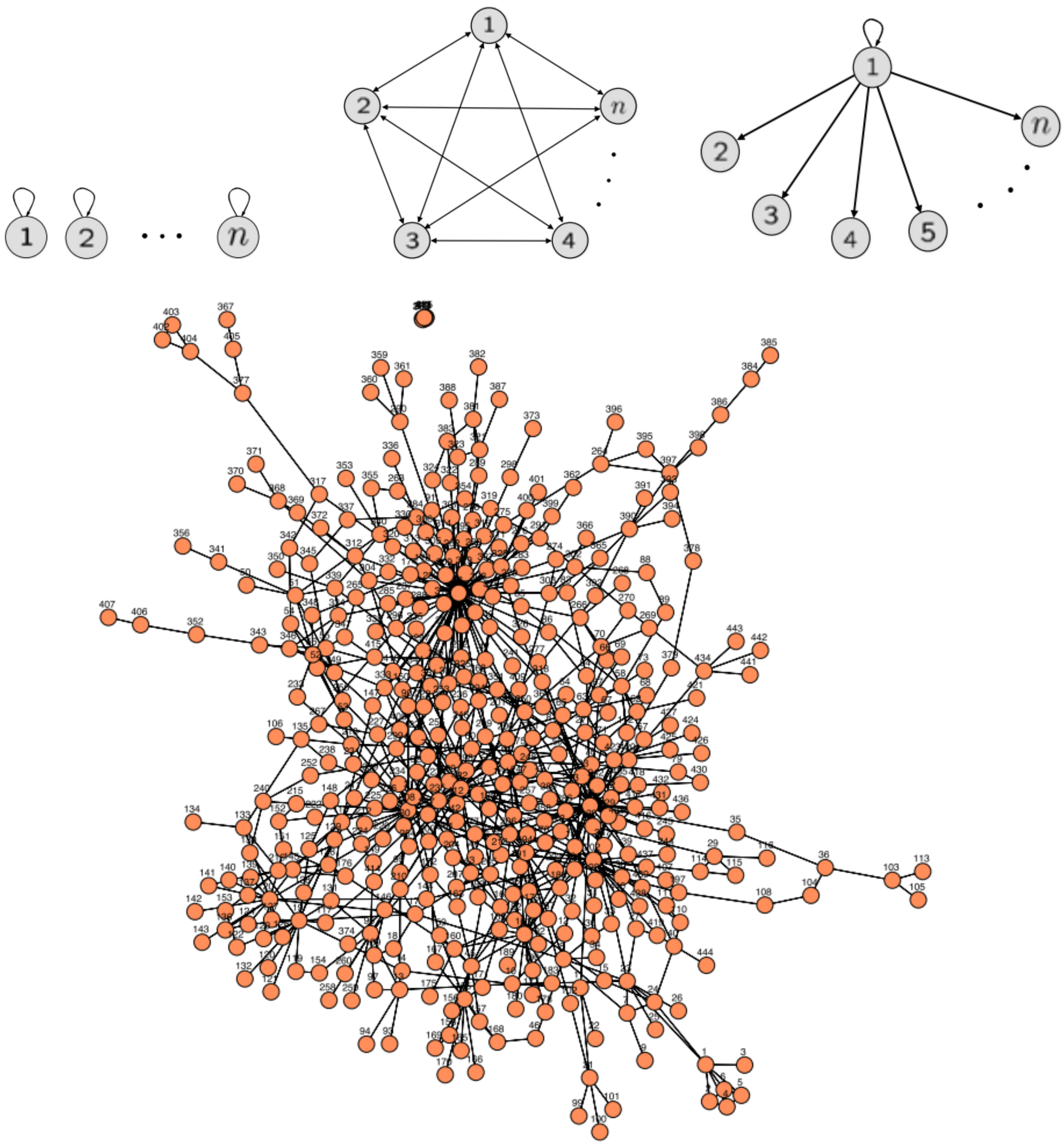
2.1 Sectoral Shocks and Aggregate Fluctuations

In their landmark contribution, J. B. Long & Plosser (1983) showed, using a simple social-planner RBC setup without frictions, government or money, how independent and serially uncorrelated shocks, via their impact on consumer preferences, smoothing and love for variety, can translate into persistence and co-movement of sectoral outputs, and the persistence of aggregate output. In their calibration to the 6-sector US economy, they also establish a disproportionate share in aggregate volatility of shocks to sectors with many productive uses, such as manufacturing and transport services. Overall they were among the first to show that business cycle phenomena were consistent with the principles of economic efficiency. J. B. Long & Plosser (1983) concluded that their model provides a good benchmark to gauge the importance of nominal frictions and other factors believed to drive business cycles. As an empirical companion to J. B. Long & Plosser (1983), B. J. B. Long & Plosser (1987) empirically investigate the monthly output series of 13 US sectors and. By performing factor analysis on the innovations from a seasonal VAR, they decompose the series into an aggregate and sectoral components. They find that a factor model with up to two aggregate factors accounts for at most 26% of the sectoral variation, and at most 47% of the variation on aggregate economy output, indicating that independent sectoral shocks and propagating mechanisms as introduced in J. B. Long & Plosser (1983) might be relevant in explaining aggregate US volatility.

The next seminal works are the papers by Horvath (1998) and Horvath (2000), intermediated by Dupor (1999). In a nutshell, Horvath (1998) calibrates a multi-sector RBC model of the US economy similar to J. B. Long & Plosser (1983), and establishes that sectoral shocks matter most of there are many sparse rows in the input-use matrix, that is if some sectors supply more inputs to more sectors than others. He shows that the 2-digit US input-use matrix is indeed sparse, and that independent sectoral shocks could account for as much as 80% of aggregate US volatility. Dupor (1999) critiques this results with a classical argument that, as the economy becomes more disaggregated, independent sectoral shocks shocks should cancel each other out and aggregate volatility from sectoral shocks should decline with the law of large numbers - that is at the rate of $\sqrt{n}$, where n is the number of sectors. In response, Horvath (2000), using a calibrated 36-sector RBC model of US economy, shows that limited interaction, characterized by a sparse input-output matrix, reduces the substitution possibilities among intermediate inputs and strengthens the co-movement in sectoral value-added (VA). This sparsity effect then leads to a postponement of the law of large numbers in bringing down the variance of aggregate value-added, which was thought by Dupor (1999) and others to thwart a determining influence if idiosyncratic sectoral shocks to aggregate fluctuations. Horvath (2000) further shows that aggregate shocks of the required magnitude and persistence are hard to observe in the data, and that his model is able to generate realistic US business cycles without relying on such aggregate shocks, while at the same time capable of providing more precise policy prescriptions.

After Horvath (2000) and the sparsity result, a further key contribution was brought in by Acemoglu et al. (2012), who theoretically showed that in the presence of inter-sectoral input-output linkages, microeconomic idiosyncratic shocks may lead to aggregate fluctuations, but that the rate at which aggregate volatility decays is determined by the structure of the network capturing such linkages, and not the sparsity of the input-output matrix as such. They find that sizeable aggregate volatility is obtained from sectoral shocks only if there exists significant asymmetries in the roles sectors play as suppliers to others. The sparseness of the input-output matrix is thereby unrelated to the nature of aggregate fluctuation. They support the mathematical results in their paper with some nice illustrations, some if which I reproduce below in Figure (1).

Figure 1: Model Network Structures and the US 3-digit Network Structure



Figures taken from Acemoglu et al. (2012)

The results of Acemoglu et al. (2012) answer the debate among Dupor (1999) and Horvath (2000) by establishing that sectoral shocks cancel out in the aggregate in economies where sectors are either self-contained or equally important as suppliers of input. This is shown in the first two network structures in Figure (1). Such shocks however propagate through the entire economy in productive networks with a few key suppliers, as shown in the third network structure in Figure (1). Below the 3 networks, Figure (1) show the US 3-digit production network. It most closely resembles the third structure, with some sectors figuring significantly more important as suppliers of inputs than others.

As a recent concerted effort, Atalay (2017) quantifies the contribution of sectoral shocks to business cycle fluctuations in aggregate US output, using data on U.S. industries input prices and input choices. His approach is a bit more rigorous than Horvath (2000): he also estimates the sectoral elasticities of substitution, and derives a model-filter from the state-space representation of his DSGE model which he then applies to the data in order to filter out the volatility attributable to sectoral shocks. He shows that goods produced by different industries are mainly complements to one another as inputs in downstream industries production functions. These complementarities indicate that industry-specific shocks are substantially more important than previously thought, accounting for at least half of aggregate volatility (Atalay (2017)'s estimate is up to 80% of aggregate US volatility).

2.2 Medium-Run Fluctuations and Endogenous Technology

Whereas long-run growth and business cycles have traditionally, and continue to be, studied as separate phenomena with dominions cleanly separated by the Hodrik-Prescott (HP) filter, it the seminal work of Comin & Gertler (2006) that introduced the profession to the idea of the medium-run cycle. In their paper, Comin & Gertler (2006) use a high-pass filter to separate out fluctuations below 200 quarters (which amounts to subtracting a very smooth non-linear trend) in US output, and characterize the remaining variation as a "medium-run cycle". They proceed to show that this cycle is substantially more volatile and persistent than conventional business-cycle measures, and features significant pro-cyclical movements in embodied and disembodied technological change, R&D, and in the efficiency and intensity of resource utilization. Comin & Gertler (2006) then present a NK-DSGE model of the medium-run cycle which incorporates decentralized endogenous R&D and technology diffusion as well as resource under-utilization, and thus fully endogenizes the movements in productivity that appear central to the persistence of these medium-run fluctuations. The medium-run cycle of Comin & Gertler (2006) is shown in Figure (2). The fluctuations around the medium-term component constitute the HP-business cycle traditionally studied.

Figure 2: Medium-Run Cycle in US Nonfarm Business Output per Person

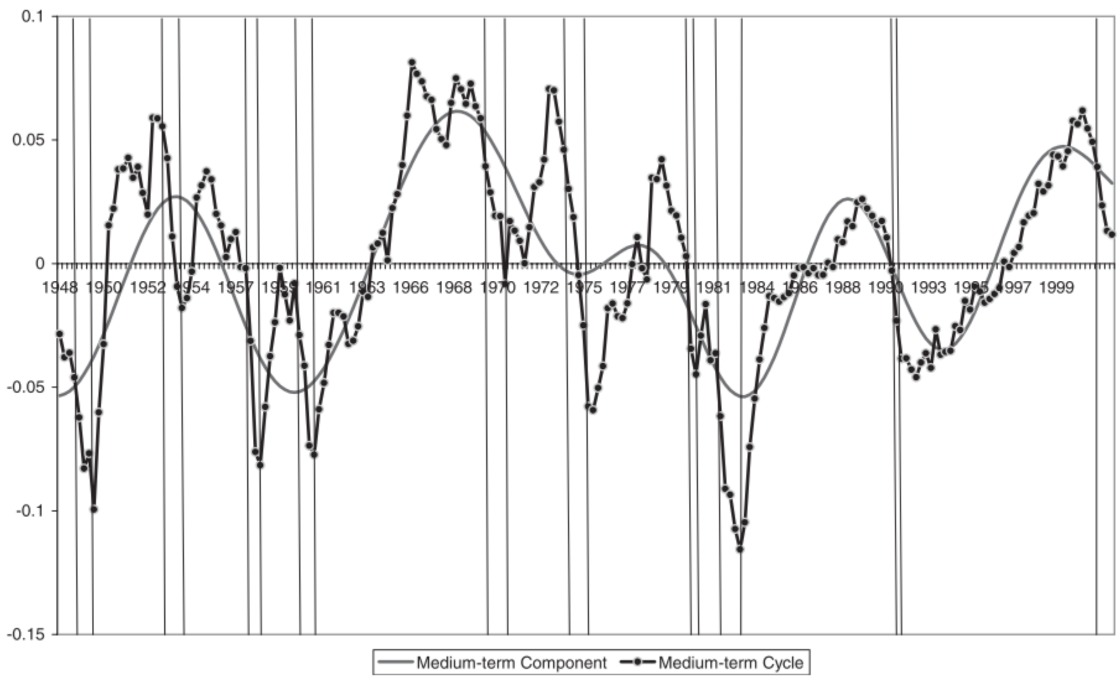


Figure taken from Comin & Gertler (2006)

Building on the work of Comin & Gertler (2006), Comin (2009) presents some empirical evidence in R&D, technology diffusion and productivity patterns affirming the relevance of macro models where endogenous technological change mechanisms are responsible both for long-run growth and the propagation of low-persistence shocks. The paper also presents a simple model of endogenous technological change and diffusion that is consistent with this evidence and easier to handle than the model of Comin & Gertler (2006). Also building on Comin & Gertler (2006), Anzoategui et al. (2017) present an even more advanced endogenous R&D and technology adoption model incorporating several types of nominal frictions. Anzoategui et al. (2017) use this model to examine the hypothesis that the slowdown in productivity following the Great Recession[2] was to a significant extent an endogenous response to the contraction in demand that induced the downturn. They begin by presenting panel data evidence that technology diffusion is highly cyclical, and calibrate the model with the endogenous TFP mechanism to the US economy. They show that the model's implied cyclicality of technology diffusion is consistent with the panel data evidence. Afterwards they use the model to assess the sources of the productivity slowdown, and show that a significant part of the fall in productivity following the Great Recession after the 2008 financial crisis was endogenous.

In light of these efforts in the literature, the model introduced in this dissertation appears very relevant and capable of generating further insights into the drivers of the medium-run cycle. It is to my knowledge the first model of this kind, integrating endogenous technology creation and adoption into a multi-sector DSGE framework.

3 A Simple Multi-Sector RBC Model

Consider a simple multi-sector RBC economy comprised of N sectors, referenced by i. In this economy, aggregate consumption is a CES aggregate of consumption goods produced by N sectors, and households supply differentiated labor to the different sectors

$$c_t = \left[\sum_{i=1}^{N} \omega_i^{\frac{1}{\epsilon}} c_{it}^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \qquad l_t = \left[\sum_{i=1}^{N} \varsigma_i^{\frac{1}{\nu}} l_{it}^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}, \tag{1}$$

where ω_i are time-invariant shares of each sectors goods in the consumers preferences, ς_i are shares determining the allocation of labor to the different sectors, and ϵ and ν are elasticities of substitution of the different sectors in the consumers consumption preferences and labor supply. The corresponding consumer price index (CPI) and aggregate wage index are given by

$$p_t = \left[\sum_{i=1}^{N} \omega_i p_{it}^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}}, \qquad w_t = \left[\sum_{i=1}^{N} \varsigma_i w_{it}^{1-\nu} \right]^{\frac{1}{1-\nu}}. \tag{2}$$

3.1 Households

I in the following represent all households by a representative household that maximizes his or her present discounted lifetime utility w.r.t. consumption and labor supply. This households present discounted lifetime utility is given by

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{l_t^{1+\varphi}}{1+\varphi} \right], \tag{3}$$

with β the inter-temporal discount factor, σ the relative risk aversion coefficient, and φ the marginal disutility from labor supply. Households also invest in an aggregate investment good that firms use to replenish their capital stocks. The price index in Eq. (2) denotes the cost of capital investment in each sector. Since little is known about depreciation rates in different sectors, I will assume an equal depreciation rate δ across all sectors, and perfect capital mobility equalizes the rental rate of capital r_t in all sectors. Assuming that the representative household own the firms, he maximize this utility function subject to an inter-temporal budget constraint. In a model without borrowing, his resource constraint stipulates that consumption and investment in each period need to be financed by wage-income, capital income and dividends

$$\sum_{i=1}^{N} (p_{it} c_{it} + p_t i_{it} - w_{it} l_{it} - r_t p_t k_{it} - \pi_{it}) = 0. \tag{4}$$

The law of motion for the aggregate capital stock is given by

$$k_{t+1} = (1-\delta) k_t + i_t. \tag{5}$$

[2] The economic downturn following the 2008/2009 global financial crisis.

Capital, investment and firm profits have simple linear aggregators

$$k_t = \sum_{i=1}^{N} k_{it}, \quad i_t = \sum_{i=1}^{N} i_{it}, \quad \pi_t = \sum_{i=1}^{N} \pi_{it}. \tag{6}$$

Now following Herrendorf et al. (2014), since only the choice of investment involves inter-temporal dynamics, the optimization problem laid down above can be broken down into one inter-temporal choice problem and 2 allocation problems. Starting with the latter, taking the aggregate consumption quantity as given, the representative household chooses c_{it} subject to a resource constraint $\sum_{i=1}^{N} p_{it} c_{it} = p_t c_t$. Simiarly, workers maximize their wage income $\sum_{i=1}^{N} w_{it} l_{it}$ subject to the CES aggregator. The outcome of these problems is described in Dixit & Stiglitz (1977) and gives a simple set of equations describing optimal consumption, and labor allocation[3]

$$c_{it} = c_t \omega_i \left(\frac{p_{it}}{p_t}\right)^{-\epsilon}; \qquad l_{it} = l_t \varsigma_i \left(\frac{w_{it}}{w_t}\right)^{\nu}. \tag{7}$$

Using the aggregators and wage/price indices, the budget constraint in Eq. (4) can be aggregated

$$p_t(c_t + i_t) = w_t l_t + r_t p_t k_t + \pi_t. \tag{8}$$

Substituting the capital accumulation rule into the budget constraint for i_t yields

$$p_t c_t + p_t k_{t+1} - p_t(1-\delta)k_t = w_t l_t + r_t p_t k_t + \pi_t. \tag{9}$$

The representative household then maximizes Eq. (3) subject to this budget constraint. The first-order conditions (FOC's) for consumption and capital yield the Euler- and labor supply equations

$$c_t^{-\sigma} = \beta E_t \left[c_{t+1}^{-\sigma}\left(1 - \delta + r_{t+1}\right)\right], \tag{10}$$

$$l_t^{\varphi} = \frac{w_t}{c_t^{\sigma} p_t}. \tag{11}$$

The Euler equation describes the representative households optimal inter-temporal consumption choice by equating the marginal utility (MU) of consumption today to the discounted MU of consumption tomorrow. The labor supply equation provides the optimal intra-temporal choice between consumption and labor by equating the relative allocation of consumption and labor to the real wage (w_t/p_t).

3.2 Firms

On the production side each sector i has a representative firm with a standard Copp-Douglas production function of the form

$$y_{it} = a_{it} k_{it}^{\alpha_i} l_{it}^{\beta_i} M_{it}^{1-\alpha_i-\beta_i} \quad \forall\, i, \tag{12}$$

with intermediate input composite

$$M_{it} = \left[\sum_{j=1}^{N} \gamma_{ji}^{\frac{1}{\eta_i}} m_{jit}^{\frac{\eta_i - 1}{\eta_i}}\right]^{\frac{\eta_i}{\eta_i - 1}} \qquad \forall\, i. \tag{13}$$

The notation followed throughout this document is that $m_{ji} = m_{\text{origin}\rightarrow\text{destiny}}$ denotes the intermediate input quantity supplied by sector j to sector i. The shares γ_{ji} correspond to the input shares of each sector, as obtained from a column-normalized input-output (IO) matrix, with η_i the sector-specific elasticity of substitution between the different intermediate inputs. Each sector further has an exogenous technology process of the form

$$\log a_{it} = (1-\rho_i)\log a_i^* + \rho_i \log a_{i,t-1} + u_{it} + v_t \quad \forall\, i, \tag{14}$$

[3] I note that the specification of ν instead of $-\nu$ in the optimal allocation of labor was made as an ad-hoc adjustment, since the problem in itself would minimize labor income just as consumption expenditure is minimized. This adjustment was considered necessary since using leisure instead of labor dramatically complicates the optimization problem, and moving beyond a CES structure for labor in a simple model like this was deemed inconvenient.

with u_{it} a sector-specific technology shock and v_t an aggregate technology shock applying equally to all sectors. In each period the representative firm in each sector chooses capital, labor and the quantity of each intermediate input to buy in order to maximize profits

$$\max_{l_{it},k_{it},m_{jit}} \quad \pi_{it} = p_{it} a_{it} k_{it}^{\alpha_i} l_{it}^{\beta_i} \left[\sum_{j=1}^{N} \gamma_{ji}^{\frac{1}{\eta_i}} m_{jit}^{\frac{\eta_i - 1}{\eta_i}} \right]^{\frac{\eta_i(1-\alpha_i-\beta_i)}{\eta_i - 1}} - w_{it} l_{it} - r_t p_t k_{it} - \sum_{j=1}^{N} p_{jt} m_{jit} \quad \forall i. \tag{15}$$

It would be possible to take into account the consumers demand curve facing each sector given by Eq. (7) to determine the monopoly price in each period, but for now I will assume that each sector is actually populated by a continuum measure unity of perfectly competitive firms which take the sectoral price of output and the sectoral wage as given, thus the representative firm acts as if it had an infinitely small share of the sectoral output. The FOC's are

$$\frac{\partial \pi_{it}}{\partial k_{it}} \Rightarrow \quad \frac{r_t p_t}{p_{it}} = \alpha_i \frac{y_{it}}{k_{it}} \quad \forall i \tag{16}$$

$$\frac{\partial \pi_{it}}{\partial l_{it}} \Rightarrow \quad \frac{w_{it}}{p_{it}} = \beta_i \frac{y_{it}}{l_{it}} \quad \forall i \tag{17}$$

$$\frac{\partial \pi_{it}}{\partial m_{jit}} \Rightarrow \quad m_{jit} = \left(\frac{p_{it}}{p_{jt}}\right)^{\eta_i} (1-\alpha_i-\beta_i)^{\eta_i} \gamma_{ji} y_{it}^{\eta_i} M_{it}^{1-\eta_i} \quad \forall i \, \forall j. \tag{18}$$

There are N^2 of Eq. (18), one per intermediate input for each sector. Now one needs to find the price of output in each sector, p_{it}. In perfect competition $P = MC$, so we need to find the MC of each sector. The total cost is given by[4]

$$TC_{it} = w_{it} l_{it} + r_t p_t k_{it} + \sum_{j=1}^{N} p_{jt} m_{jit} = p_{it} y_{it} \quad \forall i. \tag{19}$$

Inserting Eq. (18) into Eq. (13) yields, after some manipulation

$$\begin{aligned} M_{it} &= p_{it}(1-\alpha_i-\beta_i) y_{it} \underbrace{\left[\sum_{j=1}^{N} \gamma_{ji} p_{jt}^{1-\eta_i} \right]^{\frac{-1}{1-\eta_i}}}_{1/p_{M_{it}}} \\ \frac{p_{M_{it}}}{p_{it}} &= (1-\alpha_i-\beta_i) \frac{y_{it}}{M_{it}}. \end{aligned} \tag{20}$$

rewriting Eq. (19) as

$$TC_{it} = p_{it} a_{it} k_{it}^{\alpha_i} l_{it}^{\beta_i} M_{it}^{1-\alpha_i-\beta_i} \quad \forall i, \tag{21}$$

and inserting Equations (16), (17) and (20) into it gives

$$\begin{aligned} TC_{it} &= p_{it} a_{it} \left(\alpha_i \frac{y_{it} p_{it}}{r_t p_t}\right)^{\alpha_i} \left(\beta_i \frac{y_{it} p_{it}}{w_{it}}\right)^{\beta_i} \left((1-\alpha_i-\beta_i) \frac{y_{it} p_{it}}{p_{Mit}}\right)^{1-\alpha_i-\beta_i} \quad \forall i, \\ &= y_{it} p_{it}^2 a_{it} \left(\frac{\alpha_i}{r_t p_t}\right)^{\alpha_i} \left(\frac{\beta_i}{w_{it}}\right)^{\beta_i} \left(\frac{1-\alpha_i-\beta_i}{p_{Mit}}\right)^{1-\alpha_i-\beta_i} \quad \forall i. \end{aligned} \tag{22}$$

Now setting $p_{it} = MC_{it} = \frac{\partial TC_{it}}{\partial y_{it}}$ gives the ideal sectoral price of output in terms of each sectors technology and production parameters, the sectoral wage, and (as captured by the price-index for intermediate inputs p_{Mit}) the prices of output of all other sectors, weighted by their shares in sector i's production and the elasticity of substitution

$$p_{it} = \frac{1}{a_{it}} \left(\frac{r_t p_t}{\alpha_i}\right)^{\alpha_i} \left(\frac{w_{it}}{\beta_i}\right)^{\beta_i} \left(\frac{p_{Mit}}{1-\alpha_i-\beta_i}\right)^{1-\alpha_i-\beta_i} \quad \forall i. \tag{23}$$

Attentive readers will notice that Eq. (23) describes a system of N equations in $2N$ unknowns, w_i and p_i, which will provide some difficulty to solve later on. The model is closed by an equilibrium

[4] The second equality follows from the assumption of perfect competition i.e. no profits.

condition for each sector of the form

$$y_{it} = c_{it} + i_{it} + \sum_{j=1}^{N} m_{ijt} \quad \forall\, i. \tag{24}$$

The following Table (1) summarizes the equations of the model.

Table 1: Simple N-Sector RBC Model

Equation	Definition
$c_t^{-\sigma} = \beta E_t \left[c_{t+1}^{-\sigma} \left(1 - \delta + r_{t+1}\right)\right]$	Euler Equation
$l_t^{\varphi} = \frac{w_t}{c_t^{\sigma} p_t}$	labor Supply
$c_{it} = c_t \omega_i \left(\frac{p_{it}}{p_t}\right)^{-\epsilon} \quad \forall\, i$	Optimal Consumption Choice
$l_{it} = l_t \varsigma_i \left(\frac{w_{it}}{w_t}\right)^{\nu} \quad \forall\, i$	Optimal labor Allocation
$k_{t+1} = (1-\delta)k_t + i_t$	Capital Law of Motion
$y_{it} = a_{it} k_{it}^{\alpha_i} l_{it}^{\beta_i} M_{it}^{1-\alpha_i-\beta_i} \quad \forall\, i$	Production Function Sector i
$M_{it} = \left[\sum_{j=1}^{N} \gamma_{ji}^{\frac{1}{\eta_i}} m_{jit}^{\frac{\eta_i - 1}{\eta_i}}\right]^{\frac{\eta_i}{\eta_i - 1}} \quad \forall\, i$	Intermediate Inputs Sector i
$k_{it} = \alpha_i \frac{y_{it} p_{it}}{r_t p_t} \quad \forall\, i$	Demand for Capital Sector i
$l_{it} = \beta_i \frac{y_{it} p_{it}}{w_{it}} \quad \forall\, i$	Demand for labor Sector i
$m_{jit} = \left(\frac{p_{it}}{p_{jt}}\right)^{\eta_i} (1-\alpha_i-\beta_i)^{\eta_i} \gamma_{ji} y_{it}^{\eta_i} M_{it}^{1-\eta_i} \quad \forall\, i\, \forall\, j$	Demand for sector j, Sector i
$p_t = \left[\sum_{i=1}^{N} \omega_i p_{it}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$	Ideal Price Index
$w_t = \left[\sum_{i=1}^{N} \varsigma_i w_{it}^{1-\nu}\right]^{\frac{1}{1-\nu}}$	Average Wage
$p_{Mit} = \left[\sum_{j=1}^{N} \gamma_{ji} p_{jt}^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}} \quad \forall\, i$	Price of Intermediates Sector i
$p_{it} = \frac{1}{a_{it}} \left(\frac{r_t p_t}{\alpha_i}\right)^{\alpha_i} \left(\frac{w_{it}}{\beta_i}\right)^{\beta_i} \left(\frac{p_{Mit}}{1-\alpha_i-\beta_i}\right)^{1-\alpha_i-\beta_i} \quad \forall\, i$	Optimal Price Sector i
$y_{it} = c_{it} + i_{it} + \sum_{j=1}^{N} m_{ijt} \quad \forall\, i$	Equilibrium Condition Sector i
$\log a_{it} = (1-\rho_i) \log a_i^* + \rho_i \log a_{i,t-1} + u_{it} + v_t \quad \forall\, i$	Technology Shock Sector i
$k_t = \sum_{i=1}^{N} k_{it}$	Capital Aggregation
$i_t = \sum_{i=1}^{N} i_{it}$	Investment Aggregation
$y_t = \sum_{i=1}^{N} y_{it}$	Output Aggregation (Optional)

3.3 Steady State Solution

The steady state of the model is obtained by assuming that all variables in the model do not vary over time, and then solving the models equations for the values that all variables must take for this to be the case. I denote steady-state values by superscripting them with a $*$ ($x_{it} \to x_i^*$). In computing the steady state I follow the widespread convention of setting $a_i^* = 1\ \forall\, i$. The Euler Equation gives

$$r^* = \frac{1}{\beta} - (1-\delta). \tag{25}$$

The next step is to determine the prices and wages. Since all prices and wages are related, and, in a perfectly competitive equilibrium, obey Walras Law, I will apply a normalization by setting the average wage $w^* = 1$. With this normalization, the average wage index can be written as

$$1 = \sum_{i=1}^{N} \varsigma_i w_i^{*1-\nu} \quad \Rightarrow \quad w_i^* = \left(\frac{1}{\varsigma_i} - \sum_{j \neq i} \frac{\varsigma_j}{\varsigma_i} w_j^{*1-\nu}\right)^{\frac{1}{1-\nu}} \quad \forall\, i. \tag{26}$$

From the optimal sectoral prices, I yield

$$w_i^* = \beta_i (p_i^* a_i^*)^{\frac{1}{\beta_i}} \left(\frac{r^* p^*}{\alpha_i}\right)^{-\frac{\alpha_i}{\beta_i}} \left(\frac{p_{Mi}^*}{1-\alpha_i-\beta_i}\right)^{\frac{\alpha_i+\beta_i-1}{\beta_i}} \quad \forall\, i. \tag{27}$$

The pricing problem can now be solved numerically, either by taking Eq. (26) and Eq. (27) and solving a system of $2N$ equations with $2N$ unknowns (w_i^* and p_i^*), or by plugging Eq. (26) into Eq. (27), and Eq. (27) into the optimal sectoral price equation (23), and then solving a system of N equations in p_i^*, before using Eq. (27) to get the wages. With prices and wages determined, the next step is to solve the system of equilibrium conditions to get the outputs y_i^*. From the capital law of motion (which can be disaggregated) and the demand for capital

$$i_i^* = \delta k_i^* = \delta \alpha_i \frac{p_i^*}{r^* p^*} y_i^* \quad \forall i. \tag{28}$$

Combining the optimal consumption choice with the labor supply equation and the optimal labor allocation yields

$$c_i^* = \left(\frac{w^*}{l^{\varphi} p^*}\right)^{\frac{1}{\sigma}} \omega_i \left(\frac{p_i^*}{p^*}\right)^{-\epsilon} = \left(\frac{w^*}{p^*}\right)^{\frac{1}{\sigma}} \omega_i \left(\frac{p_i^*}{p^*}\right)^{-\epsilon} \left(\frac{l_i^*}{\varsigma_i}\right)^{-\frac{\varphi}{\sigma}} \left(\frac{w_i^*}{w^*}\right)^{\frac{\nu\varphi}{\sigma}} \quad \forall i. \tag{29}$$

Now inserting also the demand for labor gives

$$c_i^* = \left(\frac{w^*}{p^*}\right)^{\frac{1}{\sigma}} \omega_i \left(\frac{p_i^*}{p^*}\right)^{-\epsilon} \left(\frac{\beta_i}{\varsigma_i} \frac{p_i^*}{w_i^*}\right)^{-\frac{\varphi}{\sigma}} \left(\frac{w_i^*}{w^*}\right)^{\frac{\nu\varphi}{\sigma}} y_i^{*-\frac{\varphi}{\sigma}} \quad \forall i. \tag{30}$$

The FOC's for the intermediate goods supplied by sector i to other sectors are

$$m_{ij}^* = \left(\frac{p_j^*}{p_i^*}\right)^{\eta_j} (1-\alpha_j-\beta_j)^{\eta_j} \gamma_{ij} y_j^{*\eta_j} M_j^{*1-\eta_j} \quad \forall i\, \forall j. \tag{31}$$

These FOC's need to be re-expressed in terms of outputs and prices and then plugged into the equilibrium conditions. Dividing two of the m_{ij} yields

$$\frac{m_{kj}^*}{m_{ij}^*} = \frac{\gamma_{kj}}{\gamma_{ij}} \left(\frac{p_i^*}{p_k^*}\right)^{\eta_j} \quad \Rightarrow \quad m_{kj}^* = \frac{\gamma_{kj}}{\gamma_{ij}} \left(\frac{p_i^*}{p_k^*}\right)^{\eta_j} m_{ij}^* \quad \forall k\, \forall i\, \forall j, \tag{32}$$

and plugging Eq. (32) into the intermediate goods composite yields

$$M_j^* = \left[\sum_{k=1}^{N} \gamma_{kj} \gamma_{ij}^{\frac{1-\eta_j}{\eta_j}} \left(\frac{p_i^*}{p_k^*}\right)^{\eta_j - 1}\right]^{\frac{\eta_j}{\eta_j - 1}} m_{ij}^* \quad \forall i\, \forall j. \tag{33}$$

Now plugging this back into Eq. (31) yields

$$m_{ij}^* = \frac{p_{jt}}{p_{it}} (1-\alpha_j-\beta_j) \gamma_{ij}^{\frac{1}{\eta_j}} \left[\sum_{k=1}^{N} \gamma_{kj} \gamma_{ij}^{\frac{1-\eta_j}{\eta_j}} \left(\frac{p_i^*}{p_k^*}\right)^{\eta_j - 1}\right]^{-1} y_{jt} \quad \forall i\, \forall j. \tag{34}$$

Plugging Equations (28), (30) and (34) into the equilibrium conditions (24) gives a system of N equations in the sectoral outputs y_i^*, which also needs to be solved numerically. With outputs, wages and prices determined, all other steady-state values in the model are easily determined.

3.4 Simulation

With the steady-state determined, I let dynare compute a 1st-order Taylor expansion of the model around the steady-state, and then perform a stochastic simulation over 2000 periods (200 periods burn-in) of a stylized symmetric 2-sector version of the model. The parameters used in this simulation are largely taken from Costa (2016) and shown in Table (2).

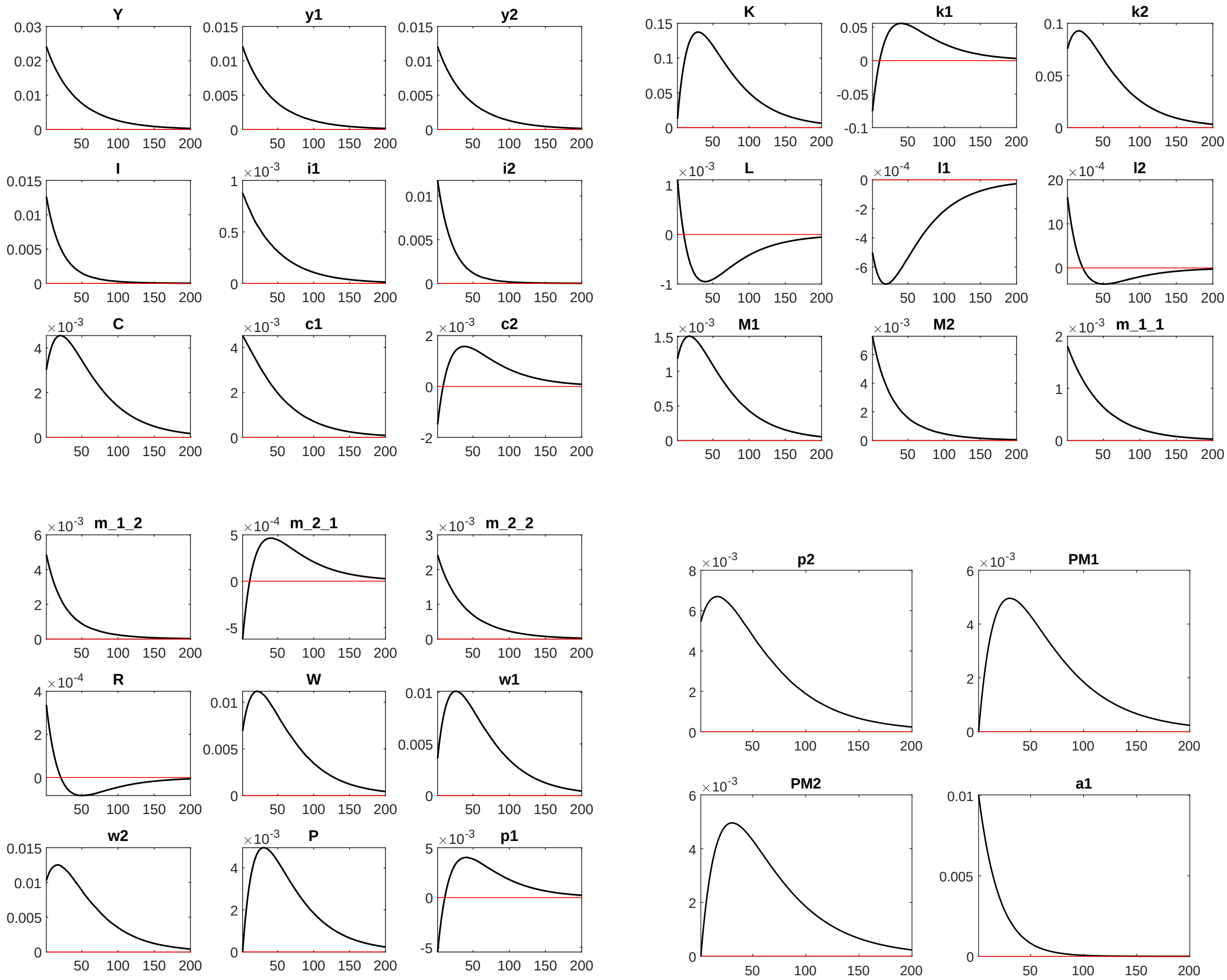
Table 2: Symmetric 2-Sector Parameterization

Parameter	Value	Parameter	Value
σ	2	φ	1.5
β	0.985	δ	0.025
ϵ	0.8	ν	0.8
η_1	0.8	η_2	0.8
ω_1	0.5	ω_2	0.5
ς_1	0.5	ς_2	0.5
α_2	0.35	α_2	0.35
β_1	0.3	β_2	0.3
γ_{11}	0.5	γ_{21}	0.5
γ_{12}	0.5	γ_{22}	0.5
ρ_1	0.95	ρ_2	0.95

Figure (3) below shows the Impulse Response Functions (IRF's) obtained from a 0.1 standard-deviation productivity shock to sector 1. It is evident that the shock causes output to increase equally in the two sectors[5]. Consumption also increases in both sectors, but is initially a lot higher in sector 1 (initially relative consumption shifts to sector 1). The shock initially decreases the capital stock in sector 1, but higher investment replenishes and increases it after 10 periods leading to an increased use of capital in both sectors for an extended period of time. The behavior of capital is mirrored by labor. The shock to productivity decreases the use of labor in sector 1, and initially increases employment in sector 2, but after a while the use of labor in both sectors is below it initial value - production has become more capital intensive. The movements in capital and labor use are matched by the change in the rental rate of capital, which initially spikes by the initial demand for capital coming triggered by the shock, but then decreases and remains below its initial value for an extended period of time as the aggregate capital stock grows. Simultaneously, the wage rate in both sectors increases and remains higher for a long time.

[5] This is only the case by virtue of the symmetric parameterization. When the model is calibrated in section 6, the responses of sectoral outputs to a shock in one sector are more heterogenous.

Figure 3: Impulse Response Functions Following 0.1 sd Shock to Sector 1



The use of intermediate inputs in production increases in both sectors following the shock, but more in sector 2 than in sector 1. At the disaggregated level, we can see that sector 1 supplies substantially more inputs to itself and sector 2, while sector 2 initially supplies less inputs to sector 1 and more to itself. This movement in inputs is triggered by a movement in relative prices as the price of sector 1 initially decreases following the shock and the price of sector 2 increases.

Having thus introduced the basic multi-sector RBC model and studied its mechanics at a basic level, I proceed to introduce a simple one-sector RBC model in which the production process is now augmented to allow for endogenous technology creation and adoption. Since technology is only privately created if there are profits, there now needs to be imperfect competition.

4 A Simple RBC Model with Endogenous R&D and Technology Diffusion

The model of production introduced in this section is adapted from Comin (2009) and Anzoategui et al. (2017) and involves a fully decentralized production and innovation process involving 4 agents: Perfectly competitive final goods (retail) firms, monopolistically competitive intermediate goods (wholesale) firms, technology adopters and technology innovators. The latter two also reap the benefits of imperfect competition in the wholesale sector by selling production plans and ideas, respectively. The benefits to the consumer from technology creation and adoption will be in terms of expanding variety, which will manifest itself in an expansion of the number of wholesale firms, each producing a single differentiated product. A distinction between technology creation and adoption is made to allow for realistic lags in the adoption process. The optimization problems of these 4 types of agents are described in turn.

4.1 Final Good (Retail) Firms

As in the textbook NK model, the final goods firm is perfectly competitive and aggregates intermediate goods produced by a continuum measure a_t, where a_t is the stock of adopted technologies and the number of wholesale firms, of wholesale firms (indexed by k) to a consumption bundle purchased by the consumer

$$y_t = \left(\int_0^{a_t} y_{kt}^{\frac{\psi-1}{\psi}} dk \right)^{\frac{\psi}{\psi-1}}. \tag{35}$$

In Eq. (35) ψ denotes the elasticity of substitution among wholesale goods from the perspective of the consumer who purchases the consumption bundle. The retail firm maximizes revenues taking aggregate and input prices as given

$$\max_{y_{kt}} \quad p_t \left(\int_0^{a_t} y_{kt}^{\frac{\psi-1}{\psi}} dk \right)^{\frac{\psi}{\psi-1}} - \int_0^{a_t} p_{kt} y_{kt} dk. \tag{36}$$

Taking the FOC w.r.t. any particular y_{kt} yields the demand function for wholesale good k, which is directly proportional to aggregate demand and inversely proportional to its relative price level

$$\begin{aligned} \frac{\psi}{\psi-1} p_t \left(\int_0^{a_t} y_{kt}^{\frac{\psi-1}{\psi}} dk \right)^{\frac{\psi}{\psi-1}-1} \frac{\psi-1}{\psi} y_{kt}^{\frac{\psi-1}{\psi}-1} - p_{kt} &= 0 \\ p_t \left(\int_0^{a_t} y_{kt}^{\frac{\psi-1}{\psi}} dk \right)^{\frac{1}{\psi-1}} y_{kt}^{\frac{-1}{\psi}} &= p_{kt} \\ y_{kt} &= y_t \left(\frac{p_t}{p_{kt}} \right)^{\psi}. \end{aligned} \tag{37}$$

Substituting the demand function back in the aggregator function yields the ideal price index, which is the CPI of the consumers consumption bundle

$$\begin{aligned} y_t &= y_t p_t^{\psi} \left(\int_0^{a_t} \left(\frac{1}{p_{kt}} \right)^{\psi-1} dk \right)^{\frac{\psi}{\psi-1}} \\ p_t &= \left(\int_0^{a_t} p_{kt}^{1-\psi} dk \right)^{\frac{1}{1-\psi}}. \end{aligned} \tag{38}$$

Since in this model all wholesale firms are identical in their pricing behavior, the integral in Eq. (38) can be solved and rewritten as

$$p_t = a_t^{\frac{1}{1-\psi}} p_{kt} \quad \text{or} \quad p_{kt} = a_t^{\frac{1}{\psi-1}} p_t. \tag{39}$$

The same is true for output, Eq. (35) can be solved and written as

$$y_t = a_t^{\frac{\psi}{\psi-1}} y_{kt} \quad \text{or} \quad y_{kt} = a_t^{\frac{\psi}{1-\psi}} y_t. \tag{40}$$

4.2 Intermediate Good (Wholesale) Firms

The representative intermediate goods firm chooses capital k_{kt} and unskilled labor l_{ukt} to produce output by the following Copp-Douglas technology

$$y_{kt} = \theta_t k_{kt}^{\alpha} l_{ukt}^{1-\alpha}, \tag{41}$$

with θ_t a stationary productivity shock to the intermediate goods sector. Wholesale firms then choose inputs and the price of their output subject to the final good firms (consumers) demand function derived in Eq. (37)

$$\max_{k_{kt},\, l_{ukt},\, p_{kt}} \quad \pi_{kt} = p_{kt} \theta_t k_{kt}^{\alpha} l_{ukt}^{1-\alpha} - r_t p_t k_{kt} - w_{ut} l_{ukt}. \tag{42}$$

Rewriting Eq. (37) yields the inverse demand function $p_{kt} = p_t \left(\frac{y_{kt}}{y_t}\right)^{\frac{-1}{\psi}} = p_t \left(\frac{\theta_t k_{kt}^{\alpha} l_{ukt}^{1-\alpha}}{y_t}\right)^{\frac{-1}{\psi}}$, and the problem becomes

$$\max_{k_{kt},\, l_{kt}} \quad \pi_{kt} = p_t y_t^{\frac{1}{\psi}} \left(\theta_t k_{kt}^{\alpha} l_{ukt}^{1-\alpha}\right)^{\frac{\psi-1}{\psi}} - r_t p_t k_{kt} - w_{ut} l_{ukt}. \tag{43}$$

Assuming that each intermediate good firm is very small w.r.t. the whole of intermediate goods firms, so that its choice of inputs does not impact the aggregate price or quantity, yields the FOC's

$$\frac{\partial \pi_{kt}}{\partial k_{kt}} = p_t y_t^{\frac{1}{\psi}} \frac{\psi-1}{\psi} \theta_t \alpha k_{kt}^{\alpha-1} l_{ukt}^{1-\alpha} \left(\theta_t k_{kt}^{\alpha} l_{ukt}^{1-\alpha}\right)^{\frac{-1}{\psi}} - r_t p_t = 0 \quad \Rightarrow \quad \underbrace{p_{kt} \alpha \frac{y_{kt}}{k_{kt}}}_{\text{MR(k)}} = \frac{\psi}{\psi-1} \underbrace{r_t p_t}_{\text{MC(k)}} \tag{44}$$

$$\frac{\partial \pi_{kt}}{\partial l_{ukt}} = p_t y_t^{\frac{1}{\psi}} \frac{\psi-1}{\psi} \theta_t (1-\alpha) k_{kt}^{\alpha} l_{ukt}^{-\alpha} \left(\theta_t k_{kt}^{\alpha} l_{ukt}^{1-\alpha}\right)^{\frac{-1}{\psi}} - w_{ut} = 0 \quad \Rightarrow \quad \underbrace{p_{kt}(1-\alpha) \frac{y_{kt}}{l_{ukt}}}_{\text{MR(l)}} = \frac{\psi}{\psi-1} \underbrace{w_{ut}}_{\text{MC(l)}}. \tag{45}$$

Inserting these FOC's back into the inverse demand function gives the optimal pricing choice of the individual wholesale firm

$$\begin{aligned} p_{kt} &= p_t y_t^{\frac{1}{\psi}} \left(\theta_t \left(p_{kt} \alpha \frac{y_{kt}}{r_t p_t} \frac{\psi-1}{\psi}\right)^{\alpha} \left(p_{kt}(1-\alpha) \frac{y_{kt}}{w_{ut}} \frac{\psi-1}{\psi}\right)^{1-\alpha}\right)^{\frac{-1}{\psi}} \\ p_{kt}^{\frac{\psi+1}{\psi}} &= p_t \left(\frac{y_{kt}}{y_t}\right)^{\frac{-1}{\psi}} \left(\theta_t \frac{\psi-1}{\psi} \left(\frac{\alpha}{r_t p_t}\right)^{\alpha} \left(\frac{1-\alpha}{w_{ut}}\right)^{1-\alpha}\right)^{\frac{-1}{\psi}} \\ p_{kt} &= \left(\theta_t \frac{\psi-1}{\psi} \left(\frac{\alpha}{r_t p_t}\right)^{\alpha} \left(\frac{1-\alpha}{w_{ut}}\right)^{1-\alpha}\right)^{-1} = \frac{\psi}{\psi-1} \underbrace{\frac{1}{\theta_t} \left(\frac{r_t p_t}{\alpha}\right)^{\alpha} \left(\frac{w_{ut}}{1-\alpha}\right)^{1-\alpha}}_{\text{MC}}. \end{aligned} \tag{46}$$

This result is well known since Dixit & Stiglitz (1977) and states that in a monopolistically competitive equilibrium featuring a continuum of identical firms, the optimal price of output is a constant markup over the marginal cost.

4.3 Technology Adopters

Wholesale products are first invented and then adopted, but following Anzoategui et al. (2017) this subsection describes their adoption conditional on their invention, before describing their invention in the next section. The adoption process considered here is pro-cyclical but takes time. It is also decentralized e.g. aggregate patterns are described without taking account of individual firm adoptions.

Assuming that in each period new technologies are created, only a fraction λ_t of these technologies become directly usable in the same period. Whether a specific technology becomes usable is thereby modelled a random draw with success probably λ_t. Once a technology is usable, all wholesale firms are able to employ it immediately, which is modelled by an expansion in the number of varieties a_t. We will think of this expansion as the adopter selling the technology to a newly created intermediate goods firm (a start-up). Pro-cyclical adoption behavior is then obtained by endogenizing the probability λ_t that a new technology becomes usable and making it increasing in the amount of resources devoted to adoption at the firm level. More formally, let z_t be the stock of invented technologies, then following Anzoategui et al. (2017), the probability $0 < \lambda_t < 1$ is given by a concave function

$$\lambda_t = \kappa (z_t l_{sat})^{\rho_a}, \tag{47}$$

where κ and $0 < \rho_a < 1$ ($\lambda' > 0,\ \lambda'' < 0$) are constants, and l_{sat} is the skilled labor investment devoted to technology adoption in each period. The presence of z_t accounts for the fact that the adoption process becomes more efficient as the technological state of the economy improves. There is also a technical need for this spillover in that it ensures a balanced growth path: As technologies grow, the number of new goods requiring adoption increases, but the supply of labor remains unchanged, so it needs to be scaled by z_t. Following this description, if $\bar{\lambda}$ is the steady state value of λ_t, then the average time it takes for a new technology be adopted is $1/\bar{\lambda}$.

Once in usable form, the adopter sells the rights to the technology to a monopolistically competitive wholesale firm, which earns a profit from employing the technology. The value to the adopter of successfully bringing a new technology into use, v_t, is therefore given by the present discounted value of the wholesale firms profits from operating the technology. Accordingly, given that r_{t+1} is the one period interest rate between $t+1$ and t, we can express, v_t, as

$$v_t = \pi_t + \phi E_t \frac{v_{t+1}}{1+r_{t+1}}, \tag{48}$$

where ϕ is the probability that the technology survives (i.e. does not become obsolete), which works like a discount factor here. Iterating yields an expected discounted value. We can then express the adopter's maximization problem as choosing l_{sat} to maximize the value J_t gained from the acquisition of unadopted technologies, given by

$$J_t = \underset{l_{sat}}{max} \quad - w_{st} l_{sat} + \phi E_t \left\{ \frac{\lambda_t v_{t+1} + (1-\lambda_t) J_{t+1}}{1+r_{t+1}} \right\}. \tag{49}$$

The first term in this Bellman equation reflects total adoption expenditures, while the second is the discounted benefit: The probability weighted sum of the values of adopted and unadopted technologies. In Eq. (49) w_{st} denotes the skilled wage paid to skilled labor engaged in adoption and R&D, which is different from the unskilled wage w_{ut} paid by wholesale firms. The FOC describing optimal adoption investment is

$$w_{st} = z_t \lambda_t' \phi E_t \left\{ \frac{v_{t+1} - J_{t+1}}{1+r_{t+1}} \right\} = \rho_a \frac{\lambda_t}{l_{sat}} \phi E_t \left\{ \frac{v_{t+1} - J_{t+1}}{1+r_{t+1}} \right\}. \tag{50}$$

Eq. (50) states that the marginal gain from adoption expenditures: the increase in the adoption success probability λ_t times the discounted difference between the value of an adopted versus an unadopted technology is equated to the marginal cost w_{st}. The term $v_{t+1} - J_{t+1}$ is pro-cyclical, by virtue of the greater influence of near term profits on the value of adopted technologies relative to unadopted ones. As a consequence, l_{sat} varies pro-cyclically, and hence the pace of adoption, given by λ_t, will also vary pro-cyclically. The description of adoption is closed by the following law of motion describing the evolution of adopted technologies

$$a_{t+1} = \lambda_t \phi [z_t - a_t] + \phi a_t, \tag{51}$$

where $z_t - a_t$ is the stock of technologies available for adoption in period t.

4.4 Technology Innovators

Analogous to the adoption sector, there is a continuum measure unity of innovators that use skilled labor to create new ideas, which adopters can then buy and transform into production plans for intermediate goods bought by wholesale firms. Let l_{srt} be skilled labor employed in R&D by the representative innovator and let ϑ_t be the marginal product of skilled labor producing a technology in a given time-period

$$\vartheta_t = \chi_t z_t l_{srt}^{\rho_z - 1}. \tag{52}$$

In this equation l_{srt} denotes the aggregate amount of skilled labor working on R&D, which an individual innovator takes as given. Following Romer (1990), the presence of z_t makes this a linear growth model. It is assumed that $0 < \rho_z < 1$, implying that increased R&D in the aggregate reduces the efficiency of R&D at the individual level. χ_t is an exogenous productivity shifter following a stochastic process

$$\log \chi_t = (1-\rho_\chi) \log \chi^* + \rho_\chi \log \chi_{t-1} + \epsilon_t^\chi. \tag{53}$$

The representative innovator chooses l_{srt} to maximize the expected value of the technology, as given by Eq. (49), which is his/her compensation

$$\underset{l_{srt}}{max} \quad E_t \frac{l_{srt} \vartheta_t J_{t+1}}{1+r_{t+1}} - w_{st} l_{srt}. \tag{54}$$

The FOC again equates the maginal discounted benefit of an additional unit of skilled labor in innovation with its marginal cost

$$E_t \frac{\vartheta_t J_{t+1}}{1+r_{t+1}} = E_t \frac{\chi_t z_t l_{srt}^{\rho_z - 1} J_{t+1}}{1+r_{t+1}} = w_{st}. \tag{55}$$

Given that profits from intermediate goods are pro-cyclical, the value of an unadopted technology, which depends on expected future profits, will also be pro-cyclical. Let ϕ again be the survival rate for any given technology. Then, we can express the evolution of technologies as

$$z_{t+1} = \phi z_t + \vartheta_t l_{srt} \quad \text{or} \quad \frac{z_{t+1}}{z_t} = \phi + \chi_t l_{srt}^{\rho_z}. \tag{56}$$

4.5 Housholds

Households consume the consumption bundle sold by the retail firm at the CPI p_t, and supply skilled and unskilled labor with labor shares ς_u, ς_s. Their objective function is given by

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{1}{\mu_{ut}\varsigma_u} \frac{l_{ut}^{1+\varphi}}{1+\varphi} - \frac{1}{\mu_{st}\varsigma_s} \frac{l_{st}^{1+\varphi}}{1+\varphi} \right]. \tag{57}$$

Following Comin (2009), μ_u and μ_s are preference-shifter shocks reflecting distortions in the labor market such as labor market frictions, labor income taxes and the like. These shocks follow stationary stochastic processes

$$\log \mu_{ut} = \rho_{\mu_u} \log \mu_{u,t-1} + \epsilon_{ut}^{\mu} \tag{58}$$

$$\log \mu_{st} = \rho_{\mu_s} \log \mu_{s,t-1} + \epsilon_{st}^{\mu}. \tag{59}$$

Households also invest in an aggregate investment good that wholesale firms use to replenish their capital stocks. Their resource constraint, in the absence of borrowing, is therefore given by

$$p_t(c_t + i_t) = w_{ut} l_{ut} + w_{st} l_{st} + r_t p_t k_t + \pi_t. \tag{60}$$

The law of motion for the capital stock is again given by

$$k_{t+1} = (1-\delta) k_t + i_t. \tag{61}$$

Substituting the capital accumulation rule into the budget constraint for i_t, yields the households inter-temporal budget constraint

$$p_t c_t + p_t k_{t+1} - p_t(1-\delta) k_t = w_{ut} l_{ut} + w_{st} l_{st} + r_t p_t k_t + \pi_t. \tag{62}$$

Following the optimization, the equations describing optimal consumption, skilled and unskilled and labor supply are given as

$$l_{ut}^{\varphi} = \varsigma_u \mu_u \frac{w_{ut}}{c_t^{\sigma} p_t} \tag{63}$$

$$l_{st}^{\varphi} = \varsigma_s \mu_s \frac{w_{st}}{c_t^{\sigma} p_t} \tag{64}$$

$$c_t^{-\sigma} = \beta E_t \left[c_{t+1}^{-\sigma} \left(1 - \delta + r_{t+1}\right)\right]. \tag{65}$$

Finally, as shown above, skilled labor is divided into skilled labor devoted to technology adoption and skilled labor devoted to R&D. Following Anzoategui et al. (2017), they are aggregated as follows

$$l_{st} = (z_t - a_t) l_{sat} + l_{srt}. \tag{66}$$

An equilibrium condition completes the description of the model

$$y_t = c_t + i_t. \tag{67}$$

4.6 Aggregation

Before the model can be simulated, the relations concerning individual intermediate goods producers and technology adopters need to be aggregated. This is done with the aggregators for output and prices in Equations (39) and (40), which I repeat below

$$y_t = a_t^{\frac{\psi}{\psi-1}} y_{kt}; \qquad p_t = a_t^{\frac{1}{1-\psi}} p_{kt}, \tag{68}$$

and the relations

$$k_t = a_t k_{kt}; \qquad l_{ut} = a_t l_{ukt}; \qquad \pi_t = a_t \pi_{kt}. \tag{69}$$

Using Eq. (46) the intermediate goods firms price can be represented in terms of the marginal cost. Performing this replacement and aggregating gives the aggregate price, and aggregate FOC's describing the behavior of the wholesale sector

$$p_t = a_t^{\frac{1}{1-\psi}} \frac{\psi}{\psi - 1} MC_t \tag{70}$$

$$MC_t \alpha \frac{y_{kt}}{k_{kt}} = r_t p_t \quad \Rightarrow \quad k_t = a_t^{\frac{1}{1-\psi}} \alpha y_t \frac{MC_t}{r_t p_t} \tag{71}$$

$$MC_t (1-\alpha) \frac{y_{kt}}{l_{ukt}} = w_{ut} \quad \Rightarrow \quad l_{ut} = a_t^{\frac{1}{1-\psi}} (1-\alpha) y_t \frac{MC_t}{w_{ut}}. \tag{72}$$

The production function and the profit equation must also be aggregated[6]

$$y_{kt} = \theta_t k_{kt}^{\alpha} l_{ukt}^{1-\alpha} \quad \Rightarrow \quad y_t = a_t^{\frac{1}{\psi-1}} \theta_t k_t^{\alpha} l_{ut}^{1-\alpha} \tag{73}$$

$$\pi_{kt} = p_{kt} \theta_t k_{kt}^{\alpha} l_{ukt}^{1-\alpha} - w_{ut} l_{ukt} - r_t p_t k_{kt} \quad \Rightarrow \tag{74}$$

$$\Pi_t = p_t a_t^{\frac{1}{\psi-1}} \theta_t k_t^{\alpha} l_{ut}^{1-\alpha} - w_{ut} l_{ut} - r_t p_t k_t \tag{75}$$

$$= p_t y_t - w_{ut} l_{ut} - r_t p_t k_t. \tag{76}$$

For technology adopters and inventors, the values of adopted and unadopted technologies and the corresponding FOC's were solved at the individual adopter/innovator level. I define the aggregate values of adopted and unadopted technologies as follows

$$v_t^a = a_t v_t; \qquad J_t^z = z_t J_t. \tag{77}$$

The relevant equations pertaining to adoption and innovation become

$$v_t^a = \Pi_t + \phi E_t \frac{v_{t+1}^a a_t}{a_{t+1}(1 + r_{t+1})} \tag{78}$$

$$J_t^z = E_t \left\{ \frac{\lambda_t v_{t+1}^a \frac{z_t}{a_{t+1}} + (1-\lambda_t) J_{t+1}^z \frac{z_t}{z_{t+1}}}{1 + r_{t+1}} \right\} - w_{st} l_{sat} z_t \tag{79}$$

$$w_{st} l_{sat} = \rho_a \lambda_t \phi E_t \left\{ \frac{\frac{v_{t+1}^a}{a_{t+1}} - \frac{J_{t+1}^z}{z_{t+1}}}{1 + r_{t+1}} \right\} \tag{80}$$

$$E_t \frac{\frac{\vartheta_t}{z_{t+1}} J_{t+1}^z}{1 + r_{t+1}} = w_{st}. \tag{81}$$

The model is now ready for simulations and summarized in Table (3).

[6] I replace π_t by Π_t to make it very explicit that the aggregate value of adopted technologies depends on aggregate profits earned in the intermediate goods sector.

Table 3: A Simple RBC Model with Endogenous Technology

Equation	Definition
$l_{ut}^{\varphi} = \varsigma_u \mu_u \frac{w_{ut}}{c_t^{\sigma} p_t}$	Unskilled labor Supply
$l_{st}^{\varphi} = \varsigma_s \mu_s \frac{w_{st}}{c_t^{\sigma} p_t}$	Skilled labor Supply
$c_t^{-\sigma} = \beta E_t \left[c_{t+1}^{-\sigma}(1-\delta+r_{t+1})\right]$	Euler Equation
$k_{t+1} = (1-\delta)k_t + i_t$	Capital Law of Motion
$y_t = a_t^{\frac{1}{\psi-1}} \theta_t k_t^{\alpha} l_{ut}^{1-\alpha}$	Production Function
$a_t^{\frac{1}{1-\psi}} \alpha \frac{y_t}{k_t} MC_t = r_t p_t$	Demand for Capital
$a_t^{\frac{1}{1-\psi}} (1-\alpha) \frac{y_t}{l_{ut}} MC_t = w_{ut}$	Demand for labor
$MC_t = \frac{1}{\theta_t} \left(\frac{r_t p_t}{\alpha}\right)^{\alpha} \left(\frac{w_{ut}}{1-\alpha}\right)^{1-\alpha}$	Marginal Cost
$a_t^{\frac{1}{\psi-1}} p_t = \frac{\psi}{\psi-1} MC_t$	(Optimal) Price Level
$\lambda_t = \kappa(z_t l_{sat})^{\rho_a}$	Adoption Success Probability
$\Pi_t = p_t a_t^{\frac{1}{\psi-1}} \theta_t k_t^{\alpha} l_{ut}^{1-\alpha} - r_t p_t k_t - w_{ut} l_{ut}$	Intermediate Goods Aggregate Profit
$v_t^a = \Pi_t + \phi E_t \frac{v_{t+1}^a a_t}{a_{t+1}(1+r_{t+1})}$	Value of Adopted Technology
$J_t^z = E_t \left\{ \frac{\lambda_t v_{t+1}^a \frac{z_t}{a_{t+1}} + (1-\lambda_t) J_{t+1}^z \frac{z_t}{z_{t+1}}}{1+r_{t+1}} \right\} - w_{st} l_{sat} z_t$	Value of Unadopted Technology
$w_{st} l_{sat} = \rho_a \lambda_t \phi E_t \left\{ \frac{\frac{v_{t+1}^a}{a_{t+1}} - \frac{J_{t+1}^z}{z_{t+1}}}{1+r_{t+1}} \right\}$	Optimal Adoption Investment
$a_{t+1} = \lambda_t \phi [z_t - a_t] + \phi a_t$	Evolution of Adopted Technology
$\vartheta_t = \chi_t z_t l_{srt}^{\rho_z - 1}$	Productivity of R&D
$E_t \frac{\frac{\vartheta_t}{z_{t+1}} J_{t+1}^z}{1+r_{t+1}} = w_{st}$	Optimal R&D Investment
$z_{t+1} = \phi z_t + \vartheta_t l_{srt}$	Evolution of Technology
$l_{st} = (z_t - a_t) l_{sat} + l_{srt}$	Skilled labor Aggregation
$y_t = c_t + i_t$	Equilibrium Condition
$\log \chi_t = (1-\rho_\chi)\log \chi^* + \rho_\chi \log \chi_{t-1} + \epsilon_t^{\chi}$	R&D Shock
$\log \theta_t = \rho_\theta \log \theta_{t-1} + \epsilon_t^{\theta}$	Productivity Shock
$\log \mu_{ut} = \rho_{\mu_u} \log \mu_{u,t-1} + \epsilon_{ut}^{\mu}$	Unskilled labor Supply Shock
$\log \mu_{st} = \rho_{\mu_s} \log \mu_{s,t-1} + \epsilon_{st}^{\mu}$	Skilled labor Supply Shock

4.7 Steady State Solution

Following Costa (2016) I normalize the price level to 1 $p^* = 1$, and of the shocks $\theta^* = \mu_u^* = \mu_s^* = 1$. In addition I set the equilibrium number of wholesale firms / adopted technologies $a^* = 1$, and following Anzoategui et al. (2017) I set the adoption success probability $\lambda^* = 0.05$.

Starting off with the prices, from the Euler Equation it follows that

$$r^* = \frac{1}{\beta} - (1-\delta), \tag{82}$$

and the optimal price equation gives

$$MC^* = \frac{\psi - 1}{\psi}. \tag{83}$$

From the marginal cost equation then

$$w_u^* = (1-\alpha) MC^{*\frac{1}{1-\alpha}} \left(\frac{r^*}{\alpha}\right)^{\frac{\alpha}{\alpha-1}}. \tag{84}$$

The demand for capital is

$$k^* = \alpha \frac{MC^*}{r^*} y^*, \tag{85}$$

and similarly the demand for labor is

$$l_u^* = (1-\alpha) \frac{MC^*}{w_u^*} y^*. \tag{86}$$

From the capital law of motion it follows that

$$i^* = \delta k^* = \delta\alpha \frac{MC^*}{r^*} y^*, \tag{87}$$

and the unskilled labor supply equation gives

$$c^* = (\varsigma_u w_u^*)^{\frac{1}{\sigma}} l_u^{-\frac{\varphi}{\sigma}} = (\varsigma_u w_u^*)^{\frac{1}{\sigma}} \left((1-\alpha)\frac{MC^*}{w_u^*}\right)^{-\frac{\varphi}{\sigma}} y^{*-\frac{\varphi}{\sigma}}. \tag{88}$$

Now inserting Equations (87) and (88) in the equilibrium condition gives

$$y^* = (\varsigma_u w_u^*)^{\frac{1}{\sigma}} \left((1-\alpha)\frac{MC^*}{w_u^*}\right)^{-\frac{\varphi}{\sigma}} y^{*-\frac{\varphi}{\sigma}} + \delta\alpha\frac{MC^*}{r^*} y^*. \tag{89}$$

$$y^* = \left(1 - \delta\alpha\frac{MC^*}{r^*}\right)^{-\frac{\sigma}{\varphi+\sigma}} \left((1-\alpha)\frac{MC^*}{w_u^*}\right)^{-\frac{\varphi}{\varphi+\sigma}} (\varsigma_u w_{ut})^{\frac{1}{\varphi+\sigma}}. \tag{90}$$

This determines the equilibrium output y^*, and from the above equations k^*, l_u^*, c^* and i^* are easily determined. Turning now to the endogenous technology part of the model, the profit equation gives

$$\Pi^* = y^* - r^* k^* - w_u^* l_u^*, \tag{91}$$

and the value of adopted technology is

$$v^{a*} = \Pi^* \frac{1}{1 - \frac{\phi}{1+r^*}}. \tag{92}$$

The evolution of adopted technology gives

$$z^* = \frac{1-\phi}{\lambda^* \phi} + 1. \tag{93}$$

Inserting the optimal adoption investment into the value of unadopted technology yields

$$J^{z*} = v^{a*} z^* \frac{1 - \rho_a \phi}{\frac{r^*}{\lambda^*} + 1 - \rho_a \phi}. \tag{94}$$

The skilled labor supply gives

$$w_s^* = \frac{l_s^{*\varphi} c^{*\sigma}}{\varsigma_s}, \tag{95}$$

and the optimal adoption investment yields

$$l_{sa}^* = \frac{\varsigma_s \rho_a \lambda^* \phi}{l_s^{*\varphi} c^{*\sigma}} \frac{v^{a*} - \frac{J^{z*}}{z^*}}{1 + r^*}. \tag{96}$$

Combining the optimal R&D investment with the evolution of technology gives

$$l_{sr}^* = \frac{\varsigma_s J^{z*}}{l_s^{*\varphi} c^{*\sigma}} \frac{1-\phi}{1+r^*}. \tag{97}$$

The aggregation of skilled labor then determines the skilled labor stock

$$l_s^* = (z^* - 1)\frac{\varsigma_s \rho_a \lambda^* \phi}{l_s^{*\varphi} c^{*\sigma}} \frac{v^{a*} - \frac{J^{z*}}{z^*}}{1 + r^*} + \frac{\varsigma_s J^{z*}}{l_s^{*\varphi} c^{*\sigma}} \frac{1-\phi}{1+r^*}. \tag{98}$$

$$l_s^* = \left(\frac{(z^* - 1)\varsigma_s \rho_a \lambda^* \phi \left(v^{a*} - \frac{J^{z*}}{z^*}\right) + \varsigma_s J^{z*}(1-\phi)}{c^{*\sigma}(1+r^*)}\right)^{\frac{1}{1+\varphi}}. \tag{99}$$

With l_s^* determined, from the above equations w_s^*, l_{sa}^* and l_{sr}^* are also determined. Now the evolution of technology yields

$$\vartheta^* = \frac{z^*(1-\phi)}{l_{sr}^*}. \tag{100}$$

Finally, the parameter κ and the value of χ^* are set to make the steady state consistent with the model. The productivity of R&D yields

$$\chi^* = \frac{\vartheta^*}{z^*} l_{sr}^{*1-\rho_z}, \tag{101}$$

and by the adoption success probability

$$\kappa = \lambda^* (z^* l_{sa}^*)^{-\rho_a}. \tag{102}$$

4.8 Simulation

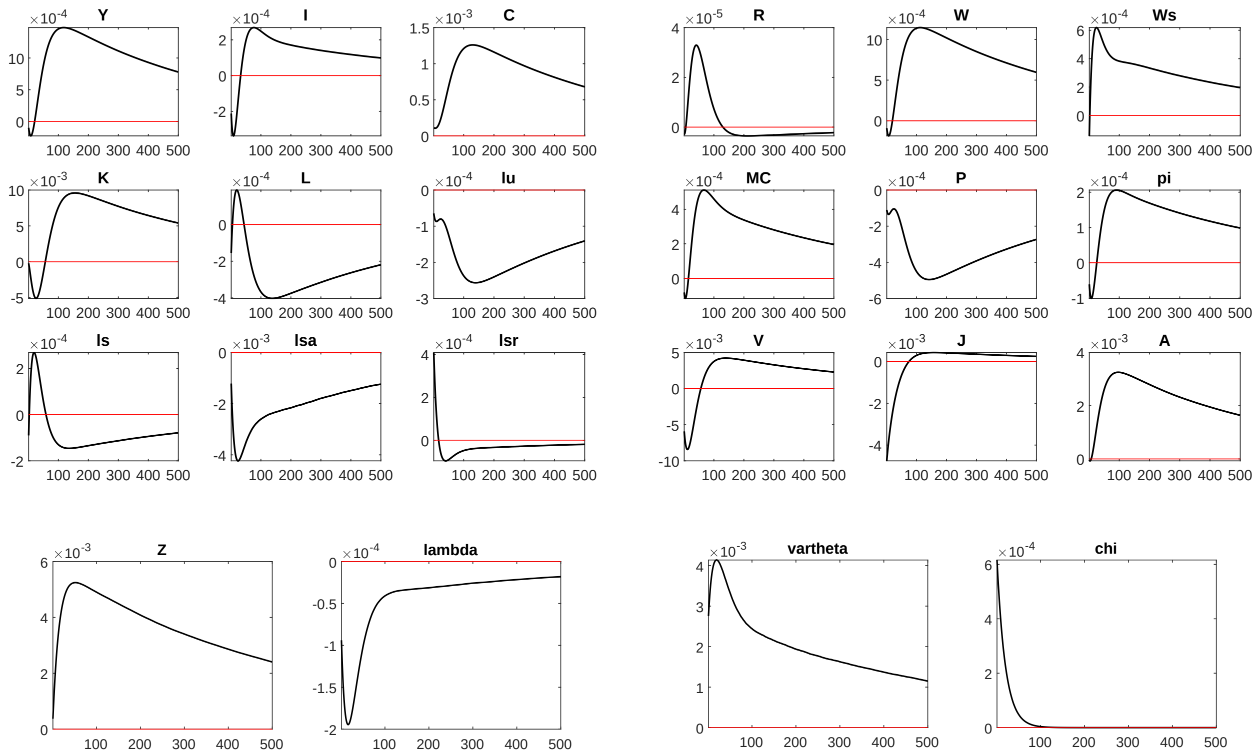
Just as with the multi-sector RBC, I let dynare compute a 1st-order Taylor Expansion of the model around the steady-state, and then perform a stochastic simulation over 2000 periods (200 periods burn-in). The parameters used in this simulation fit the US economy and are shown in Table (4).

Table 4: Parameterization a la Anzoategui et al. (2017)

Parameter	Value	Parameter	Value
σ	2	φ	1.5
ϕ	0.98	ψ	3.8571
ς_u	0.5	ς_s	0.5
α	0.3	δ	0.02
β	0.985	κ	0.0583
ρ_z	0.37	ρ_a	0.927
ρ_χ	0.95	ρ_θ	0.95
ρ_{μ_u}	0.95	ρ_{μ_s}	0.95

Figure (4) below shows the 500-Period IRF's obtained from a 0.1 standard-deviation shock to the productivity of R&D (χ). The shock initially shifts skilled labor into R&D, which comes at a slight decrease in output and investment. The shock also has permanent effects on the productivity of R%D ($vartheta$) and the stock of technologies Z, which is a consequence of the linear growth specification. After the initial jump, skilled labor in R&D quickly returns back to normal (even decreases a bit), and then the shock permanently increases output, investment, consumption and profits (pi). With a further delay, the capital stock also permanently increases and the real interest rate (R) decreases. Wages of unskilled labor (W) and skilled labor (Ws) increase permanently, and overall employment (lu and ls) decreases. The stock of adopted technologies (A), which is the number of wholesale firms, increases, consumers experience a gain from increased variety. At the same time, during the initial boom the values of adopted and unadopted technologies (V and J) decrease, but recover and increase after about 100 periods. The adoption success probability λ

Figure 4: Impulse Response Functions Following 0.1 sd R&D Shock (χ)



is permanently lower following the R&D shock. In short, a shock to the productivity of R&D in this model has little immediate impact on the economy, but prolonged effects with an extended period of increased output, consumption and technology creation and adoption following the initial boom. The peak impact of this shock is reached after 100-periods, and its effect has not died out after 500 periods. The overall response pattern resembles that of skill-biased technological change with greater consumption and gains from variety for the consumer, more capital-intensive production, higher wages for the employed, but disruptions in the labor market and an extended period of greater unemployment following the shock.

Analogous, Figure (5) below shows the 500-Period IRF's obtained from a 0.1 standard-deviation productivity shock (θ). It is immediately clear that compared to the R&D shock this shock is much more short-lived. It triggers direct increases in output, investment, consumption, capital and profits. labor input also decreases while the wage goes up, the price of output and the marginal cost decrease. On the technology side it seems like this shock decreases the stock of adopted and unadopted technologies A and Z relative to their trends.The adoption success probability λ and the productivity of R&D also decrease. In summary, the productivity shock only impacts the production side of the economy, where it leads to increases in output, consumption, investment and the capital stock, while decreasing the use of labor - similar as in the multi-sector RBC model considered in section 2. The decreased use of labor also affects skilled labor, and via skilled labor leads to a decrease in the rates of adoption and invention of new technologies. Overall this seems to be just the opposite of the pro-cyclical response of R&D and adoption expected when constructing the model. This conclusion however is erroneous, since strictly speaking in this model there should not be a productivity shock that operates independently from technology creation and adoption. For precisely this reason Comin (2009) introduces the labor supply shock as a substitute for the productivity shock. Before considering the impulse responses to this shock however I note that the responses to the productivity shock are typically a factor 10 larger than the responses to the R&D shock - thus while the R&D shock has extended positive effects, its impact is much smaller than the direct impact of the productivity shock.

Figure 5: Impulse Response Functions Following 0.1 sd Productivity Shock (θ)

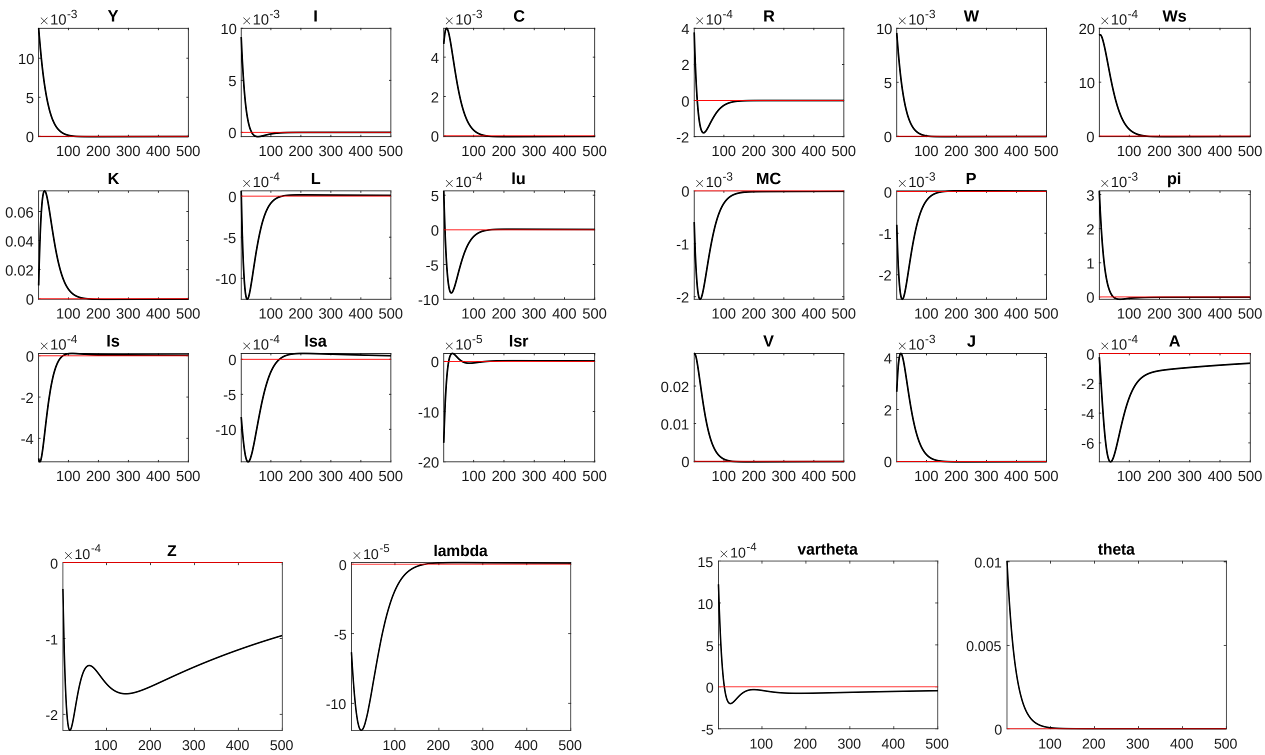
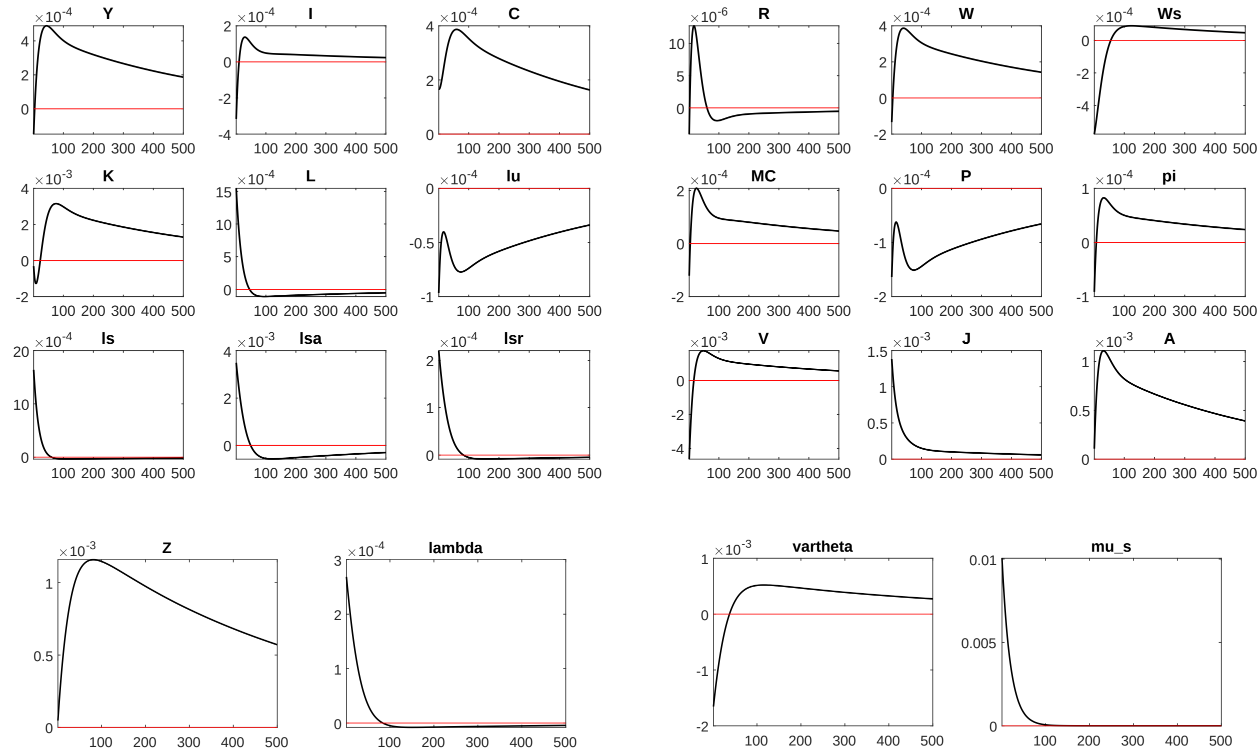


Figure (6) shows the 500-period IRF's following a skilled labor supply shock. The IRF's are similar to those from the R&D shock, but the effect in output, investment, profits and skilled wages is lower than that of the R&D shock. It is key to realize that, in contrast to the productivity shock, we here observe the pro-cyclical R&D and adoption behavior. Both the stock of adopted technologies and the stock of invented technologies and their values increase. The adoption success probability (λ_t) increases and after a while the productivity of R&D (ϑ_t) concurs. I do not report the responses for the unskilled labor supply shock, but they are very similar to those from the productivity shock i.e. the effect is much more short-lived.

Figure 6: Impulse Response Functions Following 0.1 sd Skilled Labor Supply Shock (μ_s)



5 Multi-Sector RBC with Endogenous R&D and Technology Diffusion

Having introduced and simulated both the N-sector RBC and the endogenous technology model, this section introduces an integrated N-sector RBC economy in which each sector has its own retailers, wholesale firms, technology adopters and technology innovators. While the construction of the model will involve a bit of repetition in the algebra already introduced, the novelty of the model comes in the form of very deep sectoral integration at 3 different levels: There will be intermediate inputs in wholesale firms, technology adoption spillovers which are able to propagate through the value chain from either upstream or downstream sectors, and spillovers in the productivity of R&D, which can also propagate from upstream or downstream sectors in the value chain.

5.1 Final Good (Retail) Firms

Each sector i has perfectly competitive final goods firms that aggregate intermediate goods produced by a continuum (measure a_{it}) of wholesale firms

$$y_{it} = \left(\int_0^{a_{it}} y_{kit}^{\frac{\psi_i - 1}{\psi_i}} dki \right)^{\frac{\psi_i}{\psi_i - 1}} \quad \forall i. \tag{103}$$

ψ_i is the sector-specific elasticity of substitution among wholesale goods from the perspective of the consumer who purchases the consumption bundle. The retail firm maximizes revenues taking aggregate sector and input prices as given

$$\max_{y_{kit}} \quad p_{it} \left(\int_0^{a_{it}} y_{kit}^{\frac{\psi_i - 1}{\psi_i}} dki \right)^{\frac{\psi_i}{\psi_i - 1}} - \int_0^{a_{it}} p_{kit} y_{kit} dki \quad \forall i. \tag{104}$$

Taking the FOC w.r.t. any particular y_{kit} yields the demand function for wholesale good ki, which is directly proportional to aggregate demand and inversely proportional to its relative price level

$$y_{kit} = y_{it} \left(\frac{p_{it}}{p_{kit}} \right)^{\psi_i} \quad \forall i. \tag{105}$$

Substituting the demand function back in the aggregator function yields the ideal sectoral price index

$$p_{it} = \left(\int_0^{a_{it}} p_{kit}^{1-\psi_i} dki \right)^{\frac{1}{1-\psi_i}} \quad \forall i. \tag{106}$$

Since in this model all wholesale firms are identical in their pricing behavior, the integrals for the aggregator and price index can again be solved to yield

$$y_{it} = a_{it}^{\frac{\psi_i}{\psi_i - 1}} y_{kit} \qquad \text{and} \qquad p_{it} = a_{it}^{\frac{1}{1-\psi_i}} p_{kit} \quad \forall i. \tag{107}$$

5.2 Intermediate Goods (Wholesale) Firms

The representative intermediate goods firm in sector i chooses capital k_{kit}, unskilled labor l_{ukit} and intermediate goods from other sectors (j) M_{kit} to produce output by the following Copp-Douglas technology

$$y_{kit} = \theta_{it} k_{kit}^{\alpha_i} l_{ukit}^{\beta_i} M_{kit}^{1-\alpha_i-\beta_i} \quad \forall i, \tag{108}$$

with intermediate inputs composite:

$$M_{kit} = \left[\sum_{j=1}^{N} \gamma_{ji}^{\frac{1}{\eta_i}} m_{jkit}^{\frac{\eta_i - 1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i - 1}} \quad \forall i. \tag{109}$$

The notation is $m_{ji} = m_{\text{origin}\to\text{destiny}}$. θ_{it} is a stationary productivity shock to all wholesale firms in sector i

$$\log \theta_{it} = \rho_{\theta_i} \log \theta_{i,t-1} + \epsilon_{it}^{\theta} + \epsilon \quad \forall i, \tag{110}$$

where ϵ_{it}^{θ} is a sector-specific technology shock and ϵ_t is an aggregate technology shock. The representative wholesale firm in each sector chooses inputs and the price to maximize profits in each period, subject to the final good firms (consumers) demand function

$$\begin{aligned}\max_{l_{ukit},k_{kit},m_{jkit},p_{kit}} \quad \pi_{kit} = p_{kit}\theta_{it}k_{kit}^{\alpha_i}l_{ukit}^{\beta_i}\left[\sum_{j=1}^{N}\gamma_{ji}^{\frac{1}{\eta_i}}m_{jkit}^{\frac{\eta_i-1}{\eta_i}}\right]^{\frac{\eta_i(1-\alpha_i-\beta_i)}{\eta_i-1}} \\ - w_{uit}l_{ukit} - r_t p_t k_{kit} - \sum_{j=1}^{N}p_{jt}m_{jkit} \quad \forall i.\end{aligned} \tag{111}$$

The individual firm is again assumed small, so that the unskilled wage w_{uit}, the real interest rate r_t, the aggregate price index (CPI) p_t, and price of sector j's goods p_{jt} are unaffected by the particular firms demand for inputs. Rewriting the final good firms demand function for firm k's produce yields the inverse demand function

$$p_{kit} = p_{it}\left(\frac{y_{kit}}{y_{it}}\right)^{\frac{-1}{\psi_i}} = p_{it}\left(\frac{\theta_{it}k_{kit}^{\alpha_i}l_{ukit}^{\beta_i}M_{kit}^{1-\alpha_i-\beta_i}}{y_{it}}\right)^{\frac{-1}{\psi_i}} \quad \forall i. \tag{112}$$

Inserting for p_{kit}, the problem becomes

$$\begin{aligned}\max_{l_{ukit},k_{kit},m_{jkit}} \quad \pi_{kit} = p_{it}y_{it}^{\frac{1}{\psi_i}}\left(\theta_{it}k_{kit}^{\alpha_i}l_{ukit}^{\beta_i}\left[\sum_{j=1}^{N}\gamma_{ji}^{\frac{1}{\eta_i}}m_{jkit}^{\frac{\eta_i-1}{\eta_i}}\right]^{\frac{\eta_i(1-\alpha_i-\beta_i)}{\eta_i-1}}\right)^{\frac{\psi_i-1}{\psi_i}} \\ - w_{uit}l_{ukit} - r_t p_t k_{kit} - \sum_{j=1}^{N}p_{jt}m_{jkit} \quad \forall i.\end{aligned} \tag{113}$$

The small size of the firm also guarantees that its input choices do not impact the aggregate sectoral price or quantity, yields the FOC's

$$\frac{\partial \pi_{kit}}{\partial k_{kit}} = p_{it}y_{it}^{\frac{1}{\psi_i}}\frac{\psi_i-1}{\psi_i}\alpha_i\frac{y_{kit}}{k_{kit}}y_{kit}^{\frac{-1}{\psi_i}} - r_t p_t = 0 \quad \Rightarrow \quad \underbrace{p_{kit}\alpha_i\frac{y_{kit}}{k_{kit}}}_{\text{MR(k)}} = \frac{\psi_i}{\psi_i-1}\underbrace{r_t p_t}_{\text{MC(k)}} \quad \forall i \tag{114}$$

$$\frac{\partial \pi_{kit}}{\partial l_{ukit}} = p_{it}y_{it}^{\frac{1}{\psi_i}}\frac{\psi_i-1}{\psi_i}\beta_i\frac{y_{kit}}{l_{ukit}}y_{kit}^{\frac{-1}{\psi_i}} - w_{uit} = 0 \quad \Rightarrow \quad \underbrace{p_{kit}\beta_i\frac{y_{kit}}{l_{ukit}}}_{\text{MR}(l_u)} = \frac{\psi_i}{\psi_i-1}\underbrace{w_{uit}}_{\text{MC}(l_u)} \quad \forall i \tag{115}$$

$$\frac{\partial \pi_{kit}}{\partial m_{jkit}} = p_{it}y_{it}^{\frac{1}{\psi_i}}\frac{\psi_i-1}{\psi_i}(1-\alpha_i-\beta_i)\gamma_{ji}^{\frac{1}{\eta_i}}m_{jkit}^{\frac{-1}{\eta_i}}\theta_{it}k_{kit}^{\alpha_i}l_{ukit}^{\beta_i}\left[\sum_{j=1}^{N}\gamma_{ji}^{\frac{1}{\eta_i}}m_{jkit}^{\frac{\eta_i-1}{\eta_i}}\right]^{\frac{1-\eta_i\alpha_i-\eta_i\beta_i}{\eta_i-1}} y_{kit}^{\frac{-1}{\psi_i}} - p_{jt} = 0 \tag{116}$$

$$\Rightarrow \quad \underbrace{p_{kit}(1-\alpha_i-\beta_i)\left(\frac{\gamma_{ji}}{m_{jkit}}\right)^{\frac{1}{\eta_i}}y_{kit}M_{kit}^{\frac{1-\eta_i}{\eta_i}}}_{\text{MR}(m_j)} = \frac{\psi_i}{\psi_i-1}\underbrace{p_{jt}}_{\text{MC}(m_j)} \quad \forall i. \tag{117}$$

There are $\sum_i a_i N^2$ of Eq. (117), one per intermediate input for each wholesale firm in each sector. Now one needs to find the price for each firm p_{kit}. Inserting these FOC's back into the inverse demand function gives the optimal pricing choice of the individual wholesale firm

$$p_{kit} = p_{it}y_{it}^{\frac{1}{\psi_i}}\left(\theta_{it}\left(p_{kit}\alpha_i\frac{y_{kit}}{r_t p_t}\frac{\psi_i-1}{\psi_i}\right)^{\alpha_i}\left(p_{kit}\beta_i\frac{y_{kit}}{w_{uit}}\frac{\psi_i-1}{\psi_i}\right)^{\beta_i}M_{kit}^{1-\alpha_i-\beta_i}\right)^{\frac{-1}{\psi_i}} \quad \forall i. \tag{118}$$

A problem here is that Eq. (117) depends on both M_{kit} and m_{jkit}, and all quantities need to be removed from this pricing equation. Therefore rewriting Eq. (117) yields

$$\begin{aligned} m_{jkit}^{\frac{1}{\eta_i}} &= \frac{p_{kit}}{p_{jt}}\frac{\psi_i-1}{\psi_i}(1-\alpha_i-\beta_i)\gamma_{ji}^{\frac{1}{\eta_i}}y_{kit}M_{kit}^{\frac{1-\eta_i}{\eta_i}} \quad \forall i \\ m_{jkit}^{\frac{\eta_i-1}{\eta_i}} &= \left(\frac{p_{kit}}{p_{jt}}\frac{\psi_i-1}{\psi_i}(1-\alpha_i-\beta_i)y_{kit}\right)^{\eta_i-1}\gamma_{ji}^{\frac{\eta_i-1}{\eta_i}}M_{kit}^{\frac{(1-\eta_i)(\eta_i-1)}{\eta_i}} \quad \forall i,\end{aligned} \tag{119}$$

thus

$$
\begin{aligned}
M_{kit} &= \left[\sum_{j=1}^{N} \gamma_{ji}^{\frac{1}{\eta_i}} m_{jkit}^{\frac{\eta_i-1}{\eta_i}}\right]^{\frac{\eta_i}{\eta_i-1}} = \left[\sum_{j=1}^{N} \gamma_{ji}^{\frac{1}{\eta_i}} \left(\frac{p_{kit}}{p_{jt}} \frac{\psi_i - 1}{\psi_i}(1-\alpha_i-\beta_i) y_{kit}\right)^{\eta_i - 1} \gamma_{ji}^{\frac{\eta_i-1}{\eta_i}} M_{kit}^{\frac{(1-\eta_i)(\eta_i-1)}{\eta_i}}\right]^{\frac{\eta_i}{\eta_i-1}} \quad \forall i \\
M_{kit} &= \left(p_{kit}\frac{\psi_i - 1}{\psi_i}(1-\alpha_i-\beta_i) y_{kit}\right)^{\eta_i} M_{kit}^{1-\eta_i} \left[\sum_{j=1}^{N} \gamma_{ji} p_{jt}^{1-\eta_i}\right]^{\frac{\eta_i}{\eta_i-1}} \quad \forall i \\
M_{kit} &= p_{kit}\frac{\psi_i - 1}{\psi_i}(1-\alpha_i-\beta_i) y_{kit} \underbrace{\left[\sum_{j=1}^{N} \gamma_{ji} p_{jt}^{1-\eta_i}\right]^{\frac{-1}{1-\eta_i}}}_{1/p_{M_{it}}} \quad \forall i \\
&= p_{kit}\frac{\psi_i - 1}{\psi_i}(1-\alpha_i-\beta_i) \frac{y_{kit}}{p_{M_{it}}} \quad \forall i.
\end{aligned}
\tag{120}
$$

Inserting this in Eq. (118) yields

$$
\begin{aligned}
p_{kit} &= p_{it} y_{it}^{\frac{1}{\psi_i}} \left(\theta_{it} \left(p_{kit}\alpha_i \frac{y_{kit}}{r_t p_t} \frac{\psi_i - 1}{\psi_i}\right)^{\alpha_i} \left(p_{kit}\beta_i \frac{y_{kit}}{w_{uit}} \frac{\psi_i - 1}{\psi_i}\right)^{\beta_i} \left(p_{kit}\frac{\psi_i - 1}{\psi_i}(1-\alpha_i-\beta_i)\frac{y_{kit}}{p_{M_{it}}}\right)^{1-\alpha_i-\beta_i}\right)^{\frac{-1}{\psi_i}} \\
p_{kit}^{\frac{\psi_i+1}{\psi_i}} &= \underbrace{p_{it}\left(\frac{y_{kit}}{y_{it}}\right)^{\frac{-1}{\psi_i}}}_{p_{kit}} \left(\theta_{it}\frac{\psi_i - 1}{\psi_i}\left(\frac{\alpha_i}{r_t p_t}\right)^{\alpha_i} \left(\frac{\beta_i}{w_{uit}}\right)^{\beta_i} \left(\frac{1-\alpha_i-\beta_i}{p_{M_{it}}}\right)^{1-\alpha_i-\beta_i}\right)^{\frac{-1}{\psi_i}} \quad \forall i \\
p_{kit} &= \frac{\psi_i}{\psi_i - 1} \underbrace{\frac{1}{\theta_{it}}\left(\frac{r_t p_t}{\alpha_i}\right)^{\alpha_i} \left(\frac{w_{uit}}{\beta_i}\right)^{\beta_i} \left(\frac{p_{M_{it}}}{1-\alpha_i-\beta_i}\right)^{1-\alpha_i-\beta_i}}_{MC_{it}} \quad \forall i.
\end{aligned}
\tag{121}
$$

The result is again familiar from Dixit & Stiglitz (1977), in a monopolistically competitive equilibrium the optimal price of each firm is a constant mark-up over its marginal cost.

5.3 Technology Adopters

Each sector is now populated by a continuum measure unity of technology adopters which buy ideas from the technology innovators, and convert them into production plans bought by wholesale firms. The probability that an idea can be successfully converted into a production plan in the present-period is sector specific and given by λ_i. As before, the probability $0 < \lambda_{it} < 1$ is given by a concave function

$$
\lambda_{it} = \kappa_i \left(\omega_{adi}\sum_{j=1}^{N}\gamma_{ji}a_{jt} + \omega_{aui}\sum_{j=1}^{N}\gamma_{ij}a_{jt}\right)^{\rho_{Mai}} (z_{it} l_{sait})^{\rho_{ai}} \quad \forall i, \tag{122}
$$

where κ, $0 < \rho_{Ma} < 1$ and $0 < \rho_a < 1$ are constants ($\lambda' > 0,\ \lambda'' < 0$). The first term reflects adoption learning spillovers in the sector from itself and other sectors, where the first sum reflects adoption pressures resulting from downstream sectors in the value chain (i.e. sectors that supply inputs to sector i), and the second sum reflects adoption pressures from the upstream sectors (i.e. sectors that buy sector i's output). These spillovers reflect the input-output-mix in the wholesale sector, and their intensity is regulated by ρ_{Mai}, and the weights ω_{adi} and ω_{aui} reflecting the relative importance of downstream and upstream pressures. l_{sait} is the skilled labor investment devoted to technology adoption in each period. The presence of the sector-specific technology stock z_{it} again accounts for the fact that the adoption process becomes more efficient as the technological state of the economy improves.

Apart from Eq. (122) everything is as in the one-sector model: Adopters sell the production plan to a monopolistically competitive wholesale firm, thus the value of such a production plan is the present-discounted value of the profits of a wholesale firm

$$v_{it} = \pi_{kit} + \phi_i E_t \frac{v_{i,t+1}}{1+r_{t+1}} \quad \forall\, i, \tag{123}$$

where ϕ_i is the sector-specific probability that the technology survives (i.e. does not become obsolete), which works like a discount factor. The adopter again chooses l_{sait} to maximize the value J_{it} gained from the acquisition of unadopted technologies, given by

$$J_{it} = \underset{l_{sait}}{max} \quad - w_{sit} l_{sait} + \phi_i E_t \left\{ \frac{\lambda_{it} v_{i,t+1} + (1-\lambda_{it}) J_{i,t+1}}{1+r_{t+1}} \right\} \quad \forall\, i. \tag{124}$$

The FOC describing optimal adoption investment is

$$w_{sit} = z_{it} \lambda'_{it} \phi_i E_t \left\{ \frac{v_{i,t+1} - J_{i,t+1}}{1+r_{t+1}} \right\} = \rho_{ai} \frac{\lambda_{it}}{l_{sait}} \phi_i E_t \left\{ \frac{v_{i,t+1} - J_{i,t+1}}{1+r_{t+1}} \right\} \quad \forall\, i, \tag{125}$$

and the evolution of adopted technologies is

$$a_{i,t+1} = \lambda_{it} \phi_i [z_{it} - a_{it}] + \phi_i a_{it} \quad \forall\, i, \tag{126}$$

where $z_{it} - a_{it}$ is the stock of sector-specific technologies available for adoption.

5.4 Technology Innovators

Each sector also has a continuum measure unity of innovators that use skilled labor to create new intermediate goods. Let l_{srit} be skilled labor employed in R&D by the representative innovator in sector i and let ϑ_{it} be the marginal product of skilled labor producing a technology in a given time-period

$$\vartheta_{it} = \chi_{it} z_{it} \left(\omega_{rdi} \sum_{j \neq i} \gamma_{ji} z_{jt} + \omega_{rui} \sum_{j \neq i} \gamma_{ij} z_{jt} \right)^{\rho_{Mri}} l_{srit}^{\rho_{zi}-1} \quad \forall\, i. \tag{127}$$

l_{srit} here represents the aggregate amount of skilled labor working on R&D, which an individual innovator takes as given, and $0 < \rho_{zi} < 1$, implying that increased R&D in the aggregate reduces the efficiency of R&D at the individual level. Also $\rho_{Mri} < 1$, so that there are diminishing returns to upstream or downstream innovation for the sectors own innovation process. χ_{it} is a sector-specific exogenous productivity shifter following a stochastic process

$$\log \chi_{it} = (1-\rho_{\chi i}) \log \chi_i^* + \rho_{\chi i} \log \chi_{i,t-1} + \epsilon_{it}^{\chi} \quad \forall\, i. \tag{128}$$

The representative innovator chooses l_{srit} to maximize the expected value of the technology, as given by Eq. (124)

$$\underset{l_{srit}}{max} \quad E_t \frac{l_{srit} \vartheta_{it} J_{i,t+1}}{1+r_{t+1}} - w_{sit} l_{srit} \quad \forall\, i. \tag{129}$$

The FOC again equates the maginal discounted benefit of an additional unit if skilled labor in innovation with its marginal cost

$$E_t \frac{\vartheta_{it} J_{i,t+1}}{1+r_{t+1}} = w_{sit} \quad \forall\, i. \tag{130}$$

Given that profits from intermediate goods are pro-cyclical, the value of an unadopted technology, which depends on expected future profits, will be also be pro-cyclical. Let ϕ_i again be the survival rate for any given technology. Then, we can express the evolution of technologies as

$$z_{i,t+1} = \phi_i z_{it} + \vartheta_{it} l_{srit} \quad \forall\, i. \tag{131}$$

5.5 Housholds

Households consume the consumption bundle sold by the representative retail firm in each sector at the CPI p_{it}, and supply skilled and unskilled labor to each sector. Households also invest in an aggregate investment good that wholesale firms of all sectors use to replenish their capital stocks. Aggregate consumption is a CES aggregate of consumption goods produced by N sectors, skilled labor l_{st} and unskilled labor l_{ut} are CES aggregates of sectoral skilled and unskilled labor stocks

$$c_t = \left[\sum_{i=1}^{N} \omega_i^{\frac{1}{\epsilon}} c_{it}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}, \qquad l_t = l_{ut} + l_{st}, \tag{132}$$

$$l_{ut} = \left[\sum_{i=1}^{N} \varsigma_{ui}^{\frac{1}{\nu_u}} l_{uit}^{\frac{\nu_u-1}{\nu_u}}\right]^{\frac{\nu_u}{\nu_u-1}}, \qquad l_{st} = \left[\sum_{i=1}^{N} \varsigma_{si}^{\frac{1}{\nu_s}} l_{sit}^{\frac{\nu_s-1}{\nu_s}}\right]^{\frac{\nu_s}{\nu_s-1}}. \tag{133}$$

Skilled labor in each sector is again divided into skilled labor used for technology adoption and skilled labor used for R&D. Following Anzoategui et al. (2017), this allocation is endogenously determined, by the adoption gap $z_{it} - a_{it}$

$$l_{sit} = (z_{it} - a_{it})l_{sait} + l_{srit} \quad \forall\, i. \tag{134}$$

I will assume again that all CES-shares are time-invariant. A representative household again maximizes lifetime utility w.r.t. consumption and labor supply, given by

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{1}{\mu_{ut}\varsigma_u}\frac{l_{ut}^{1+\varphi}}{1+\varphi} - \frac{1}{\mu_{st}\varsigma_s}\frac{l_{st}^{1+\varphi}}{1+\varphi}\right] \quad \forall\, i, \tag{135}$$

where β is the intertemporal discount factor, σ is the relative risk aversion coefficient, and φ is the marginal disutility w.r.t. labor supply. Assuming that households own the firms, they maximize this utility function subject to the intertemporal budget constraint. Following Comin (2009), with μ_{ut} and μ_{st} preference shifter shocks are introduced to shock the labor supply. These shocks can also be interpreted as capturing frictions in the labor market and taxes. The shocks follow stationary stochastic processes

$$\log \mu_{ut} = \rho_{\mu_u} \log \mu_{u,t-1} + \epsilon_t^{\mu_u}, \tag{136}$$

$$\log \mu_{st} = \rho_{\mu_s} \log \mu_{s,t-1} + \epsilon_t^{\mu_s}. \tag{137}$$

The price index is

$$p_t = \left[\sum_{i=1}^{N} \omega_i p_{it}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}, \tag{138}$$

and denotes the cost of capital investment in each sector. Since capital is fully mobile between sectors, there is one real interest rate r_t. In a model without borrowing, households resource constraint therefore stipulates that consumption and investment in each period need to be financed by wage-income, capital income and dividends

$$\sum_{i=1}^{N} (p_{it}c_{it} + p_t i_{it} - w_{uit}l_{uit} + w_{sit}l_{sit} - r_t p_t k_{it} - \pi_{it}) = 0. \tag{139}$$

The law of motion for the aggregate capital stock is given by

$$k_{t+1} = (1-\delta)k_t + i_t. \tag{140}$$

Since capital is fully mobile, there is also one economy-wide depreciation rate δ. Capital, investment and firm profits have simple linear aggregators

$$k_t = \sum_{i=1}^{N} k_{it}, \quad i_t = \sum_{i=1}^{N} i_{it}, \quad \pi_t = \sum_{i=1}^{N} \pi_{it}. \tag{141}$$

Following again Herrendorf et al. (2014), the optimization problem can be broken down into one intertemporal choice problem and 3 allocation problems. Starting with the latter, taking the aggregate consumption quantity as given, the representative household chooses c_{it} subject to a resource constraint $\sum_{i=1}^{N} p_{it}c_{it} = p_t c_t$. Similarly, skilled and unskilled workers maximize their wage income from skilled wage $\sum_{i=1}^{N} w_{sit}l_{sit}$ and unskilled wage $\sum_{i=1}^{N} w_{uit}l_{uit}$ subject to the aggregators. The outcomes of these problems give a simple set of equations describing optimal consumption, skilled and unskilled labor allocation[7]

$$c_{it} = c_t\omega_i\left(\frac{p_{it}}{p_t}\right)^{-\epsilon}; \qquad l_{uit} = l_{ut}\varsigma_{ui}\left(\frac{w_{uit}}{w_{ut}}\right)^{\nu_u}; \qquad l_{sit} = l_{st}\varsigma_{si}\left(\frac{w_{sit}}{w_{st}}\right)^{\nu_s}, \tag{142}$$

with optimal wage indices:

$$w_{ut} = \left[\sum_{i=1}^{N}\varsigma_{ui}w_{uit}^{1-\nu_u}\right]^{\frac{1}{1-\nu_u}}; \qquad w_{st} = \left[\sum_{i=1}^{N}\varsigma_{si}w_{sit}^{1-\nu_s}\right]^{\frac{1}{1-\nu_s}}. \tag{143}$$

Using the aggregators and wage/price indices, the budget constraint in Eq. (139) can be aggregated

$$p_t(c_t + i_t) = w_{ut}l_{ut} + w_{st}l_{st} + r_t p_t k_t + \pi_t. \tag{144}$$

Substituting the capital accumulation rule into the budget constraint for i_t, yields

$$p_t c_t + p_t k_{t+1} - p_t(1-\delta)k_t = w_{ut}l_{ut} + w_{st}l_{st} + r_t p_t k_t + \pi_t. \tag{145}$$

Maximizing Eq. (135) subject to this budget constraint yields the following equations describing optimal aggregate behavior

$$l_{ut}^{\varphi} = \varsigma_u\mu_u\frac{w_{ut}}{c_t^{\sigma}p_t} \tag{146}$$

$$l_{st}^{\varphi} = \varsigma_s\mu_s\frac{w_{st}}{c_t^{\sigma}p_t} \tag{147}$$

$$c_t^{-\sigma} = \beta E_t\left[c_{t+1}^{-\sigma}\left(1-\delta+r_{t+1}\right)\right]. \tag{148}$$

The model is closed with a set of equilibrium conditions, one for each sector

$$y_{it} = c_{it} + i_{it} + \sum_{j=1}^{N} m_{ijt} \quad \forall\, i. \tag{149}$$

5.6 Aggregation

Before the model can be simulated, the relations concerning individual intermediate goods producers and technology adopters in each sector again need to be aggregated. This is done with the aggregators for output and prices in Eq. (107)

$$y_{it} = a_{it}^{\frac{\psi_i}{\psi_i - 1}} y_{kit}; \qquad p_{it} = a_{it}^{\frac{1}{1-\psi_i}} p_{kit} \quad \forall\, i, \tag{150}$$

and the relations

$$k_{it} = a_{it}k_{kit}; \qquad l_{uit} = a_{it}l_{ukit}; \qquad m_{jit} = a_{it}m_{jkit}; \qquad M_{it} = a_{it}M_{kit}; \qquad \pi_{it} = a_{it}\pi_{kit}. \tag{151}$$

Using Eq. (121) the intermediate goods firms price can further be represented in terms of the marginal cost. Doing this replacement and aggregating gives the aggregate FOC's describing the behavior of the wholesale sectors

$$MC_{it}\alpha_i\frac{y_{kit}}{k_{kit}} = r_t p_t \quad \Rightarrow \quad k_{it} = a_{it}^{\frac{1}{1-\psi_i}}\alpha_i y_{it}\frac{MC_{it}}{r_t p_t} \quad \forall\, i \tag{152}$$

$$MC_{it}\beta_i\frac{y_{kit}}{l_{ukit}} = w_{uit} \quad \Rightarrow \quad l_{uit} = a_{it}^{\frac{1}{1-\psi_i}}\beta_i y_{it}\frac{MC_{it}}{w_{uit}} \quad \forall\, i \tag{153}$$

$$MC_{it}(1-\alpha_i-\beta_i)\left(\frac{\gamma_{ji}}{m_{jkit}}\right)^{\frac{1}{\eta_i}} y_{kit}M_{kit}^{\frac{1-\eta_i}{\eta_i}} = p_{jt} \quad \Rightarrow \tag{154}$$

$$m_{jit} = a_{it}^{\frac{\eta_i}{1-\psi_i}}(1-\alpha_i-\beta_i)^{\eta_i}y_{it}^{\eta_i}\left(\frac{MC_{it}}{p_{jt}}\right)^{\eta_i}\gamma_{ji}M_{it}^{1-\eta_i} \quad \forall\, i. \tag{155}$$

[7] Again for the labor allocation problems, I replaced $-\nu$ by ν to get the right behaviour $\rightarrow$ the sector with the relatively higher wage gets supplied more labor.

The production function and the profit equation must also be aggregated

$$y_{kit} = \theta_{it} k_{kit}^{\alpha_i} l_{ukit}^{\beta_i} M_{kit}^{1-\alpha_i-\beta_i} \quad \Rightarrow \quad y_{it} = a_{it}^{\frac{1}{\psi_i - 1}} \theta_{it} k_{it}^{\alpha_i} l_{uit}^{\beta_i} M_{it}^{1-\alpha_i-\beta_i} \quad \forall i \tag{156}$$

$$\pi_{kit} = p_{kit}\theta_{it} k_{kit}^{\alpha_i} l_{ukit}^{\beta_i} M_{kit}^{1-\alpha_i-\beta_i} - w_{uit} l_{ukit} - r_t p_t k_{kit} - \sum_{j=1}^{N} p_{jt} m_{jkit} \quad \forall i \quad \Rightarrow \tag{157}$$

$$\Pi_{it} = p_{it} a_{it}^{\frac{1}{\psi_i - 1}} \theta_{it} k_{it}^{\alpha_i} l_{uit}^{\beta_i} M_{it}^{1-\alpha_i-\beta_i} - w_{uit} l_{uit} - r_t p_t k_{it} - \sum_{j=1}^{N} p_{jt} m_{jit} \quad \forall i \tag{158}$$

$$= p_{it} y_{it} - w_{uit} l_{uit} - r_t p_t k_{it} - \sum_{j=1}^{N} p_{jt} m_{jit} \quad \forall i. \tag{159}$$

For technology adopters and inventors, the values of adopted and unadopted technologies and the corresponding FOC's were solved at the individual adopter/innovator level. I define the aggregate sectoral values of adopted and unadopted technologies as follows

$$v_{it}^{a} = a_{it} v_{it}; \qquad J_{it}^{z} = z_{it} J_{it} \quad \forall i. \tag{160}$$

The equations then become

$$v_{it}^{a} = \Pi_{it} + \phi_i E_t \frac{v_{i,t+1}^{a} a_{it}}{a_{i,t+1}(1 + r_{t+1})} \quad \forall i \tag{161}$$

$$J_{it}^{z} = E_t \left\{ \frac{\lambda_{it} v_{i,t+1}^{a} \frac{z_{it}}{a_{i,t+1}} + (1 - \lambda_{it}) J_{i,t+1}^{z} \frac{z_{it}}{z_{i,t+1}}}{1 + r_{t+1}} \right\} - w_{sit} l_{sait} z_{it} \quad \forall i \tag{162}$$

$$w_{sit} l_{sait} = \rho_{ai} \lambda_{it} \phi_i E_t \left\{ \frac{\frac{v_{i,t+1}^{a}}{a_{i,t+1}} - \frac{J_{i,t+1}^{z}}{z_{i,t+1}}}{1 + r_{t+1}} \right\} \quad \forall i \tag{163}$$

$$E_t \frac{\frac{\vartheta_{it}}{z_{i,t+1}} J_{i,t+1}^{z}}{1 + r_{t+1}} = w_{sit} \quad \forall i. \tag{164}$$

The model is now completed and summarized in Table (5).

Table 5: N-Sector RBC Model with Endogenous R&D and Technology Diffusion

Equation	Definition
$l_t = l_{ut} + l_{st}$	labor Aggregation (Optional)
$l_{ut}^{\varphi} = \varsigma_u \mu_u \frac{w_{ut}}{c_t^{\sigma} p_t}$	Unskilled labor Supply
$l_{st}^{\varphi} = \varsigma_s \mu_s \frac{w_{st}}{c_t^{\sigma} p_t}$	Skilled labor Supply
$c_t^{-\sigma} = \beta E_t[c_{t+1}^{-\sigma}(1-\delta+r_{t+1})]$	Euler Equation
$c_{it} = c_t \omega_i \left(\frac{p_{it}}{p_t}\right)^{-\epsilon} \quad \forall\, i$	Optimal Consumption Choice
$l_{uit} = l_{ut} \varsigma_{ui} \left(\frac{w_{uit}}{w_{ut}}\right)^{\nu_u} \quad \forall\, i$	Optimal Unskilled labor Allocation
$l_{sit} = l_{st} \varsigma_{si} \left(\frac{w_{sit}}{w_{st}}\right)^{\nu_s} \quad \forall\, i$	Optimal Skilled labor Allocation
$w_{ut} = \left[\sum_{i=1}^N \varsigma_{ui} w_{uit}^{1-\nu_u}\right]^{\frac{1}{1-\nu_u}}$	Average Unskilled Wage Rate
$w_{st} = \left[\sum_{i=1}^N \varsigma_{si} w_{sit}^{1-\nu_s}\right]^{\frac{1}{1-\nu_s}}$	Average Skilled Wage Rate
$k_{t+1} = (1-\delta)k_t + i_t$	Capital Law of Motion
$y_{it} = a_{it}^{\frac{1}{\psi_i - 1}} \theta_{it} k_{it}^{\alpha_i} l_{uit}^{\beta_i} M_{it}^{1-\alpha_i-\beta_i} \quad \forall\, i$	Production Function Sector i
$M_{it} = \left[\sum_{j=1}^N \gamma_{ji}^{\frac{1}{\eta_i}} m_{jit}^{\frac{\eta_i - 1}{\eta_i}}\right]^{\frac{\eta_i}{\eta_i - 1}} \quad \forall\, i$	Intermediate Inputs Sector i
$k_{it} = a_{it}^{\frac{1}{1-\psi_i}} \alpha_i y_{it} \frac{MC_{it}}{r_t p_t} \quad \forall\, i$	Demand for Kapital Sector i
$l_{uit} = a_{it}^{\frac{1}{1-\psi_i}} \beta_i y_{it} \frac{MC_{it}}{w_{uit}} \quad \forall\, i$	Demand for labor Sector i
$m_{jit} = a_{it}^{\frac{\eta_i}{1-\psi_i}} (1-\alpha_i-\beta_i)^{\eta_i} y_{it}^{\eta_i} \left(\frac{MC_{it}}{p_{jt}}\right)^{\eta_i} \gamma_{ji} M_{it}^{1-\eta_i} \quad \forall\, i \,\forall\, j$	Demand for sector j, Sector i
$p_t = \left[\sum_{i=1}^N \omega_i p_{it}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$	Ideal Price Index
$p_{M_{it}} = \left[\sum_{j=1}^N \gamma_{ji} p_{jt}^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}} \quad \forall\, i$	Price of Intermediates Sector i
$MC_{it} = \frac{1}{\theta_{it}} \left(\frac{r_t p_t}{\alpha_i}\right)^{\alpha_i} \left(\frac{w_{uit}}{\beta_i}\right)^{\beta_i} \left(\frac{p_{M_{it}}}{1-\alpha_i-\beta_i}\right)^{1-\alpha_i-\beta_i} \quad \forall\, i$	Marginal Cost Sector i
$p_{it} = a_{it}^{\frac{1}{1-\psi_i}} \frac{\psi_i}{\psi_i - 1} MC_{it} \quad \forall\, i$	(Optimal) Price Level Sector i
$\lambda_{it} = \kappa_i \left(\omega_{adi} \sum_{j=1}^N \gamma_{ji} a_{jt} + \omega_{aui} \sum_{j=1}^N \gamma_{ij} a_{jt}\right)^{\rho_{Mai}} (z_{it} l_{sait})^{\rho_{ai}} \quad \forall\, i$	Adoption Success Probability Sector i
$\Pi_{it} = p_{it} y_{it} - w_{uit} l_{uit} - r_t p_t k_{it} - \sum_{j=1}^N p_{jt} m_{jit} \quad \forall\, i$	Intermediate Goods Aggregate Profit Sector i
$v_{it}^a = \Pi_{it} + \phi_i E_t \frac{v_{i,t+1}^a a_{it}}{a_{i,t+1}(1+r_{t+1})} \quad \forall\, i$	Value of Adopted Technology Sector i
$J_{it}^z = E_t \left\{ \frac{\lambda_{it} v_{i,t+1}^a \frac{z_{it}}{a_{i,t+1}} + (1-\lambda_{it}) J_{i,t+1}^z \frac{z_{it}}{z_{i,t+1}}}{1+r_{t+1}} \right\} - w_{sit} l_{sait} z_{it} \quad \forall\, i$	Value of Unadopted Technology Sector i
$w_{sit} l_{sait} = \rho_{ai} \lambda_{it} \phi_i E_t \left\{ \frac{\frac{v_{i,t+1}^a}{a_{i,t+1}} - \frac{J_{i,t+1}^z}{z_{i,t+1}}}{1+r_{t+1}} \right\} \quad \forall\, i$	Optimal Adoption Investment Sector i
$a_{i,t+1} = \lambda_{it} \phi_i [z_{it} - a_{it}] + \phi_i a_{it} \quad \forall\, i$	Evolution of Adopted Technology Sector i
$\vartheta_{it} = \chi_{it} z_{it} \left(\omega_{rdi} \sum_{j \neq i} \gamma_{ji} z_{jt} + \omega_{rui} \sum_{j \neq i} \gamma_{ij} z_{jt}\right)^{\rho_{Mri}} l_{srit}^{\rho_{zi} - 1} \quad \forall\, i$	Productivity of R&D sector i
$E_t \frac{\frac{\vartheta_{it}}{z_{i,t+1}} J_{i,t+1}^z}{1+r_{t+1}} = w_{sit} \quad \forall\, i$	Optimal R&D Investment Sector i
$z_{i,t+1} = \phi_i z_{it} + \vartheta_{it} l_{srit} \quad \forall\, i$	Evolution of Technology Sector i
$l_{sit} = (z_{it} - a_{it}) l_{sait} + l_{srit} \quad \forall\, i$	Skilled labor Aggregation Sector i
$y_{it} = c_{it} + i_{it} + \sum_{j=1}^N m_{ijt} \quad \forall\, i$	Equilibrium Condition Sector i
$\log \chi_{it} = (1-\rho_{\chi_i}) \log \chi_i^* + \rho_{\chi_i} \log \chi_{i,t-1} + \epsilon_{it}^{\chi} \quad \forall\, i$	R&D Shock Sector i
$\log \theta_{it} = \rho_{\theta_i} \log \theta_{i,t-1} + \epsilon_{it}^{\theta} + \epsilon_t \quad \forall\, i$	Productivity Shock Sector i
$\log \mu_{ut} = \rho_{\mu_u} \log \mu_{u,t-1} + \epsilon_t^{\mu_u}$	Unskilled labor Supply Shock
$\log \mu_{st} = \rho_{\mu_s} \log \mu_{s,t-1} + \epsilon_t^{\mu_s}$	Skilled labor Supply Shock
$k_t = \sum_{i=1}^N k_{it}$	Capital Aggregation
$i_t = \sum_{i=1}^N i_{it}$	Investment Aggregation
$y_t = \sum_{i=1}^N y_{it}$	Output Aggregation (Optional)

5.7 Steady State Solution

To solve for the steady state I normalize the steady state level of adopted technology (firms) in each sector $a_i^* = 1\ \forall i$, but I will keep these a_i^* in the steady-state equations in case asymmetric stocks of adopted technology are beneficial for calibrations of the model. For the shocks $\theta_i^* = \mu_u^* = \mu_s^* = 1\ \forall i$, while χ_i^* needs to be calibrated for each sector. Following Anzoategui et al. (2017) further the adoption success probability λ_i^* is set to a value in each sector. In the simulation I will assume $\lambda_i^* = 0.05\ \forall i$.

Starting off with the prices, from the Euler Equation it follows that

$$r^* = \frac{1}{\beta} - (1-\delta). \tag{165}$$

The next step is to determine the prices and wages. Since all prices and wages are related, and, in a perfectly competitive equilibrium obey Walras Law, I will again apply a normalization by setting the average unskilled wage $w_u^* = 1$. With this normalization, the average unskilled wage can be written as

$$1 = \sum_{i=1}^{N} \varsigma_{ui} w_{ui}^{*1-\nu_u} \quad \Rightarrow \quad w_{ui}^* = \left(\frac{1}{\varsigma_{ui}} - \sum_{j \neq i} \frac{\varsigma_{uj}}{\varsigma_{ui}} w_{uj}^{*1-\nu_u} \right)^{\frac{1}{1-\nu_u}} \quad \forall i. \tag{166}$$

Inserting the optimal sectoral price into the marginal cost equation for MC_i^* yields

$$w_i^* = \beta_i \left(p_i^* a_{it}^{\frac{1}{1-\psi_i}} \frac{\psi_i}{\psi_i - 1} \theta_i^* \right)^{\frac{1}{\beta_i}} \left(\frac{r^* p^*}{\alpha_i} \right)^{-\frac{\alpha_i}{\beta_i}} \left(\frac{p_{Mi}^*}{1-\alpha_i-\beta_i} \right)^{\frac{\alpha_i+\beta_i-1}{\beta_i}} \quad \forall i. \tag{167}$$

The pricing problem can now be solved numerically, either by taking Eq. (166) and Eq. (167) and solving a system of $2N$ equations with $2N$ unknowns (w_i^* and p_i^*), or by plugging Eq. (167) into the optimal sectoral price equation, solving a system of N equations in p_i^* and then using Eq. (167) to get the wages. With prices and wages determined, the next step is to solve the system of equilibrium conditions to get the outputs y_i^*. The demand for capital is

$$k_i^* = \frac{a_i^{*\frac{1}{1-\psi_i}} \alpha_i MC_i^*}{r^* p^*} y_i^* \quad \forall i, \tag{168}$$

and similarly the demand for labor

$$l_{ui}^* = \frac{a_i^{*\frac{1}{1-\psi_i}} \beta_i MC_i^*}{w_{ui}^*} y_i^* \quad \forall i. \tag{169}$$

From the capital law of motion, which can be disaggregated, it follows that

$$i_i^* = \delta k_i^* = \delta \frac{a_i^{*\frac{1}{1-\psi_i}} \alpha_i MC_i^*}{r^* p^*} y_i^* \quad \forall i. \tag{170}$$

Combining the optimal consumption choice with the unskilled labor supply equation and the optimal unskilled labor allocation yields

$$c_i^* = \left(\frac{\varsigma_u w_u^*}{l_u^{\varphi} p^*} \right)^{\frac{1}{\sigma}} \omega_i \left(\frac{p_i^*}{p^*} \right)^{-\epsilon} = \left(\frac{\varsigma_u w_u^*}{p^*} \right)^{\frac{1}{\sigma}} \omega_i \left(\frac{p_i^*}{p^*} \right)^{-\epsilon} \left(\frac{l_{ui}^*}{\varsigma_{ui}} \right)^{-\frac{\varphi}{\sigma}} \left(\frac{w_{ui}^*}{w_u^*} \right)^{\frac{\nu_u \varphi}{\sigma}} \quad \forall i. \tag{171}$$

Now inserting also the demand for labor gives

$$c_i^* = \left(\frac{\varsigma_u w_u^*}{p^*} \right)^{\frac{1}{\sigma}} \omega_i \left(\frac{p_i^*}{p^*} \right)^{-\epsilon} \left(a_i^{*\frac{1}{1-\psi_i}} \frac{\beta_i}{\varsigma_{ui}} \frac{MC_i^*}{w_{ui}^*} \right)^{-\frac{\varphi}{\sigma}} \left(\frac{w_{ui}^*}{w_u^*} \right)^{\frac{\nu_u \varphi}{\sigma}} y_i^{*-\frac{\varphi}{\sigma}} \quad \forall i. \tag{172}$$

The FOC's for the intermediate Goods supplied by sector i to other sectors are

$$m_{ijt} = a_{jt}^{\frac{\eta_j}{1-\psi_j}} (1-\alpha_j-\beta_j)^{\eta_j} y_{jt}^{\eta_j} \left(\frac{MC_{jt}}{p_{it}} \right)^{\eta_j} \gamma_{ij} M_{jt}^{1-\eta_j} \quad \forall i\, \forall j, \tag{173}$$

with

$$M_{jt} = \left[\sum_{k=1}^{N} \gamma_{kj}^{\frac{1}{\eta_j}} m_{kjt}^{\frac{\eta_j - 1}{\eta_j}} \right]^{\frac{\eta_j}{\eta_j - 1}} \qquad \forall j. \tag{174}$$

These FOC's need to be rewritten in terms of outputs and prices and then inserted into the equilibrium conditions. Dividing two of the m_{kj} yields

$$\frac{m_{k1jt}}{m_{k2jt}} = \frac{\gamma_{k1j}}{\gamma_{k2j}} \left(\frac{p_{k2t}}{p_{k1t}} \right)^{\eta_j} \quad \Rightarrow \quad m_{k1jt} = \frac{\gamma_{k1j}}{\gamma_{k2j}} \left(\frac{p_{k2t}}{p_{k1t}} \right)^{\eta_j} m_{k2jt} \quad \forall j \, \forall k. \tag{175}$$

letting $k1 = k$ and $k2 = i$ and plugging Eq. (175) into Eq. (174) yields

$$M_{jt} = \left[\sum_{k=1}^{N} \gamma_{kj} \gamma_{ij}^{\frac{1-\eta_j}{\eta_j}} \left(\frac{p_{it}}{p_{kt}} \right)^{\eta_j - 1} \right]^{\frac{\eta_j}{\eta_j - 1}} m_{ijt} \quad \forall j \, \forall i. \tag{176}$$

Now plugging this back into Eq. (173) yields

$$m_{ijt} = a_{jt}^{\frac{\eta_j}{1-\psi_j}} (1 - \alpha_j - \beta_j)^{\eta_j} y_{jt}^{\eta_j} \left(\frac{MC_{jt}}{p_{it}} \right)^{\eta_j} \gamma_{ij} \left[\sum_{k=1}^{N} \gamma_{kj} \gamma_{ij}^{\frac{1-\eta_j}{\eta_j}} \left(\frac{p_{it}}{p_{kt}} \right)^{\eta_j - 1} \right]^{\frac{\eta_j(1-\eta_j)}{\eta_j - 1}} m_{ijt}^{1-\eta_j} \tag{177}$$

$$m_{ijt} = a_{jt}^{\frac{1}{1-\psi_j}} (1 - \alpha_j - \beta_j) y_{jt} \gamma_{ij}^{\frac{1}{\eta_j}} \frac{MC_{jt}}{p_{it}} \left[\sum_{k=1}^{N} \gamma_{kj} \gamma_{ij}^{\frac{1-\eta_j}{\eta_j}} \left(\frac{p_{it}}{p_{kt}} \right)^{\eta_j - 1} \right]^{-1} \qquad \forall j \, \forall i. \tag{178}$$

Plugging Equations (170), (172) and (178) into the equilibrium condition gives a system of N equations in the sectoral outputs y_i^*, which also needs to be solved numerically

$$y_i^* = c_i^* + i_i^* + \sum_{j=1}^{N} m_{ij}^* \quad \forall i. \tag{179}$$

With outputs, wages and prices determined, c_i^*, i_i^*, k_i^*, m_{ij}^* and l_{ui}^* are also determined by the above equations. Now the profit equation gives

$$\Pi_i^* = p_i^* y_i^* - r^* p^* k_i^* - w_{ui}^* l_{ui}^* - \sum_{j=1}^{N} p_j^* m_{ji}^* \quad \forall i, \tag{180}$$

and the value of adopted technology is

$$v_i^{a*} = \Pi_i^* \frac{1}{1 - \frac{\phi_i}{1+r^*}} \quad \forall i. \tag{181}$$

The evolution of adopted technology gives

$$z_i^* = \left(\frac{1 - \phi_i}{\lambda_i^* \phi_i} + 1 \right) a_i^* \quad \forall i. \tag{182}$$

Inserting the optimal adoption investment into the value of unadopted technology gives

$$J_i^{z*} = v_i^{a*} \frac{z_i^*}{a_i^*} \frac{1 - \rho_{ai} \phi_i}{\frac{r^*}{\lambda_i^*} + 1 - \rho_{ai} \phi_i} \quad \forall i. \tag{183}$$

Inserting the skilled labor supply into the optimal unskilled labor allocation gives

$$w_{si}^* = \left(\frac{l_{si}^*}{l_s^* \varsigma_{si}} \right)^{\frac{1}{\nu_s}} w_s = \left(\frac{l_{si}^*}{l_s^* \varsigma_{si}} \right)^{\frac{1}{\nu_s}} \frac{l_s^{*\varphi} c^{*\sigma} p^*}{\varsigma_s} = \left(\frac{l_{si}^*}{\varsigma_{si}} \right)^{\frac{1}{\nu_s}} \frac{c^{*\sigma} p^*}{\varsigma_s} l_s^{*-\frac{1}{\nu_s}+\varphi}, \tag{184}$$

with

$$l_s^* = \left[\sum_{i=1}^{N} \varsigma_{si}^{\frac{1}{\nu_s}} l_{si}^{*\frac{\nu_s - 1}{\nu_s}} \right]^{\frac{\nu_s}{\nu_s - 1}}. \tag{185}$$

The optimal adoption investment yields

$$l_{sai}^* = \frac{\rho_{ai}\lambda_i^*\phi_i}{w_{si}^*}\frac{\frac{v_i^{a*}}{a_i^*} - \frac{J_i^{z*}}{z_i^*}}{1+r^*}. \tag{186}$$

Combining the optimal R&D investment with the evolution of technology gives

$$l_{sri}^* = \frac{J_i^{z*}}{w_{si}^*}\frac{1-\phi_i}{1+r^*}. \tag{187}$$

Now inserting Eq. (185) into Eq. (184), Eq. (184) into Equations (186) and (187), and Equations (186) and (187) into the aggregation of skilled labor

$$l_{si}^* = (z_i^* - a_i^*)l_{sai}^* + l_{sri}^*, \tag{188}$$

gives a system of N equations with N unknowns, which can be solved numerically for the l_{si}^*. With l_{si}^* determined, w_{si}^*, l_{sai}^* and l_{sri}^* are also determined. Now the evolution of technology yields

$$\vartheta_i^* = \frac{z_i^*(1-\phi_i)}{l_{sri}^*}, \tag{189}$$

and the productivity of R&D yields

$$\chi_i^* = \frac{\vartheta_i^*}{z_i^*}\left(\omega_{rdi}\sum_{j\neq i}\gamma_{ji}z_j^* + \omega_{rui}\sum_{j\neq i}\gamma_{ij}z_j^*\right)^{-\rho_{Mri}} l_{sri}^{*1-\rho_{zi}}. \tag{190}$$

Finally, the parameter κ is determined by the adoption success probability to make the steady state consistent with the model[8]

$$\kappa_i = \lambda_i^*\left(\omega_{adi}\sum_{j=1}^{N}\gamma_{ji}a_j^* + \omega_{aui}\sum_{j=1}^{N}\gamma_{ij}a_j^*\right)^{-\rho_{Mai}} (z_i^* l_{sai}^*)^{-\rho_{ai}}. \tag{191}$$

5.8 Simulation

With a stylized 2-sector version of the model, I let dynare compute a 1st-order Taylor Expansion of the model around the steady-state, and then perform a stochastic simulation over 2000 periods (200 periods burn-in). The parameters used in this simulation are:

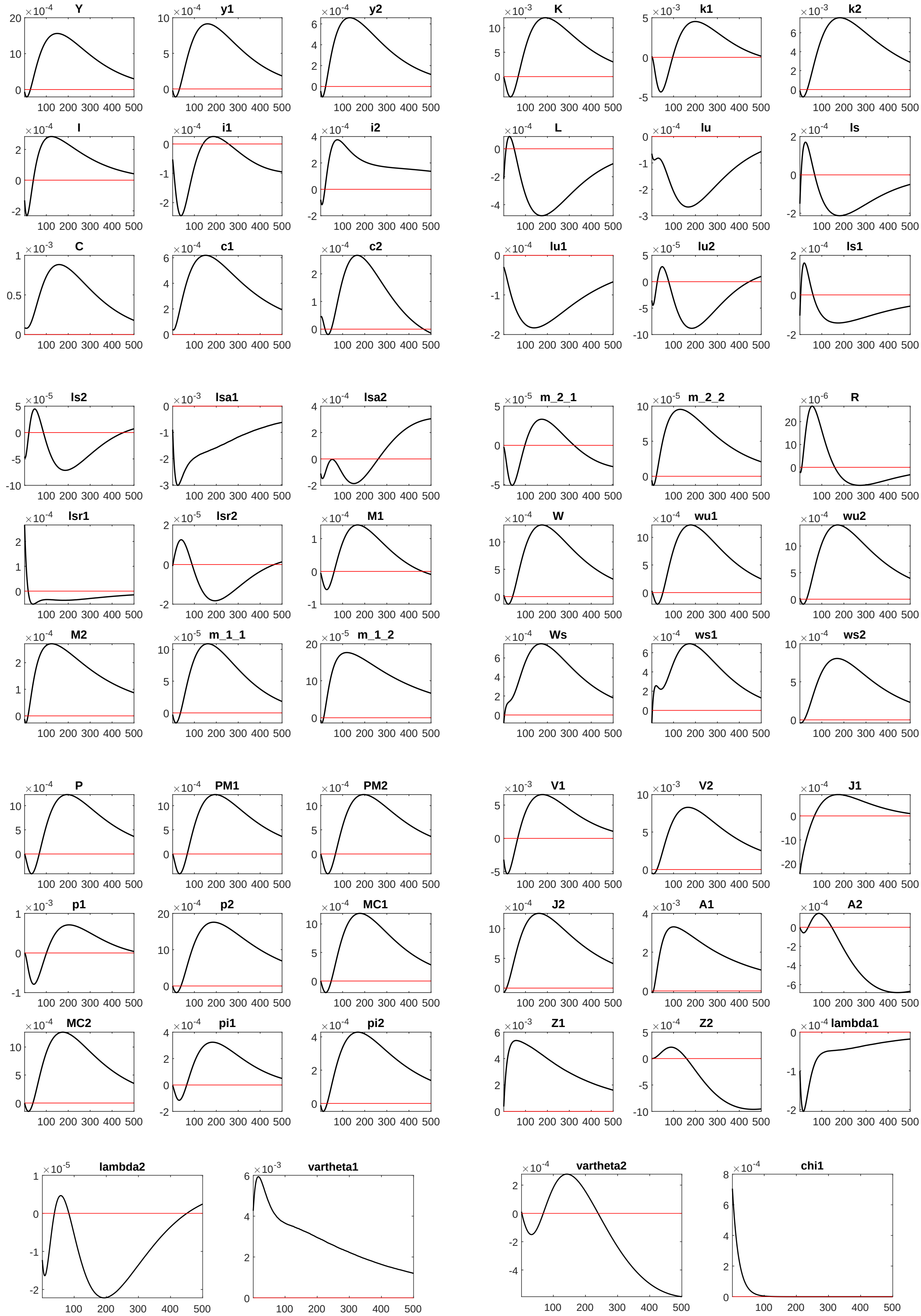
[8] If all $a_i^* = 1$, the adoption spillover term disappears.

Table 6: Parameterization a la Anzoategui et al. (2017)

Parameter	Value	Parameter	Value
σ	2	φ	1.5
β	0.985	δ	0.02
α_1	0.35	α_2	0.35
β_1	0.3	β_2	0.3
ϵ	0.8		
ν_u	0.8	ν_s	0.8
η_1	0.8	η_2	0.8
ω_1	0.5	ω_2	0.5
ω_{au1}	0.5	ω_{au2}	0.5
ω_{ad1}	0.5	ω_{ad2}	0.5
ω_{ru1}	0.5	ω_{ru2}	0.5
ω_{rd1}	0.5	ω_{rd2}	0.5
ς_u	0.5	ς_s	0.5
ς_{u1}	0.5	ς_{u2}	0.5
ς_{s1}	0.5	ς_{s2}	0.5
γ_{11}	0.5	γ_{21}	0.5
γ_{12}	0.5	γ_{22}	0.5
ϕ_1	0.98	ϕ_2	0.98
ψ_1	3.8571	ψ_2	3.8571
ρ_{a1}	0.927	ρ_{a2}	0.927
ρ_{z1}	0.37	ρ_{z2}	0.37
ρ_{Ma1}	0	ρ_{Ma2}	0
ρ_{Mr1}	0	ρ_{Mr2}	0
ρ_{χ_1}	0.95	ρ_{χ_2}	0.95
ρ_{θ_1}	0.95	ρ_{θ_2}	0.95
ρ_{μ_u}	0.95	ρ_{μ_s}	0.95
κ_1	0.082	κ_2	0.082

Note that in this parameterization $\rho_{Mai} = \rho_{Mri} = 0\ \forall\, i$, so adoption and R&D spillovers are initially disabled. Figure (7) below shows the 500-Period Impulse Response Functions (IRF's) obtained from a 0.1 standard-deviation shock to the productivity of R&D in sector 1 (χ_1). The shock increases the productivity of R&D in sector 1, and the stocks of invented and asopted technologies in sector 1. As in the one-sector model, during the adoption boom the adoption success probability in sector 1 decreases. Sector 2 displays the opposite response: The stocks of invented and adopted technology decrease vis-a-vis their previous trend, and the productivity of R&D also decreases after some curious oscillation. On the nominal side of things, the price of output in sector one initially decreases, but starts increasing after about 100 periods, while the price in sector 2 immediately goes up. The skilled and unskilled wages un both sectors increase, which is mirrored by an overall decline in the use of labor. In sector 1 the skilled wage immediately reponds to the shock, triggered by a spike in the use of skilled labor for R&D at the time of the shock. Up to this point the pattern observed in the IRF's is what could reasonably be expected from a combination of the two models previously considered. On the real side of the economy, the aggregate variables also respond to the shock in the same way as in the one-sector model, but there are some surprises concerning the relative response of the two sectors. It is curious that sector 2 seems to reap overall more benefits in terms of output, investment, capital accumulation and profits than sector 1. Only the consumption benefits of sector 1 are greater than in sector 2. Following the shock, sector 1 initially uses less intermediate inputs, and overall the supply of inputs from sector 1 to sector 2 increases asymmetrically. These results suggest the following interpretation: In a two sector set-up, the R&D shock to sector 1 lets this sector concentrate on R&D and adoption, while initially neglecting the real economy and shifting economic activity to sector 2. Overall the economy benefits from the shock, its effects on the real economy are beneficial for both sectors, and the pattern is one of skill-biased technological change as in the one-sector model, but the only immediate gain from the shock realized in sector 1 is an increase in consumption and variety. For future research, it would be interesting to see whether the introduction of frictions such as price-

Figure 7: Impulse Response Functions Following 0.1 sd R&D Shock (χ_1)



and wage stickiness or investment adjustment costs would change the observed sectoral responses and the distribution of gains from the shock in some critical respect.

In Figure (8), I repeat the R&D shock, but now allow for some R&D spillovers $\rho_{Mri} = 0.2 \; \forall \, i$, and some slight adoption spillovers $\rho_{Mai} = 0.1 \; \forall \, i$[9]. It is evident that the model is now on the verge of non-stationarity. The spillovers in R&D and adoption induce prolonged effects on output, incestment, consumption, profits and the stocks of adopted and unadopted technologies. The overall reponse pattern however is similar as in Figure (7). Notable differences to Figure (7) are the productivity of R&D in sector 2, which also permanently increased thanks to the spillover from sector 1, and the responses of prices and profits, which now permanently decreased in both sectors (for which I have no good explanation).

For the sake of completeness, in Figure (9) I show the IRF's from a 1 sd productivity shock to sector 1. As expected the effect is larger but dies out much quicker compared to the R&D shock. Just like in the one-sector model, the shock shifts resources out of R&D and adoption and into the production-side of the economy. Again however this shock is not really interesting because it occurs independently from any movements in R&D and technology adoption, which is unrealistic.

[9] I would like to have greater adoption than R&D spillovers, but it turns out that this leads to stability problems with the model which I was unable to solve in short time.

Figure 8: Impulse Response Functions Following 0.1 sd R&D Shock (χ_1) + Spillovers

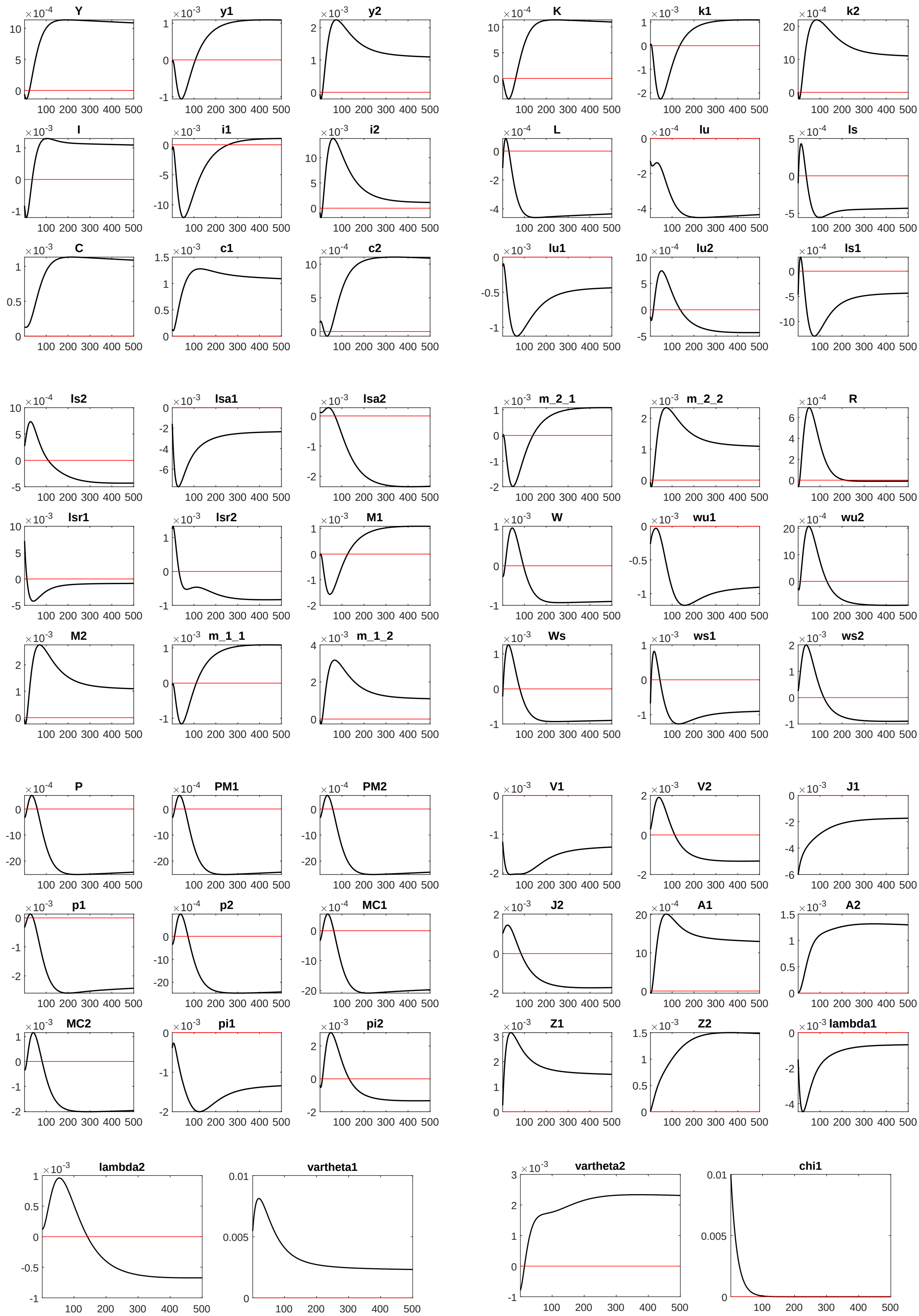
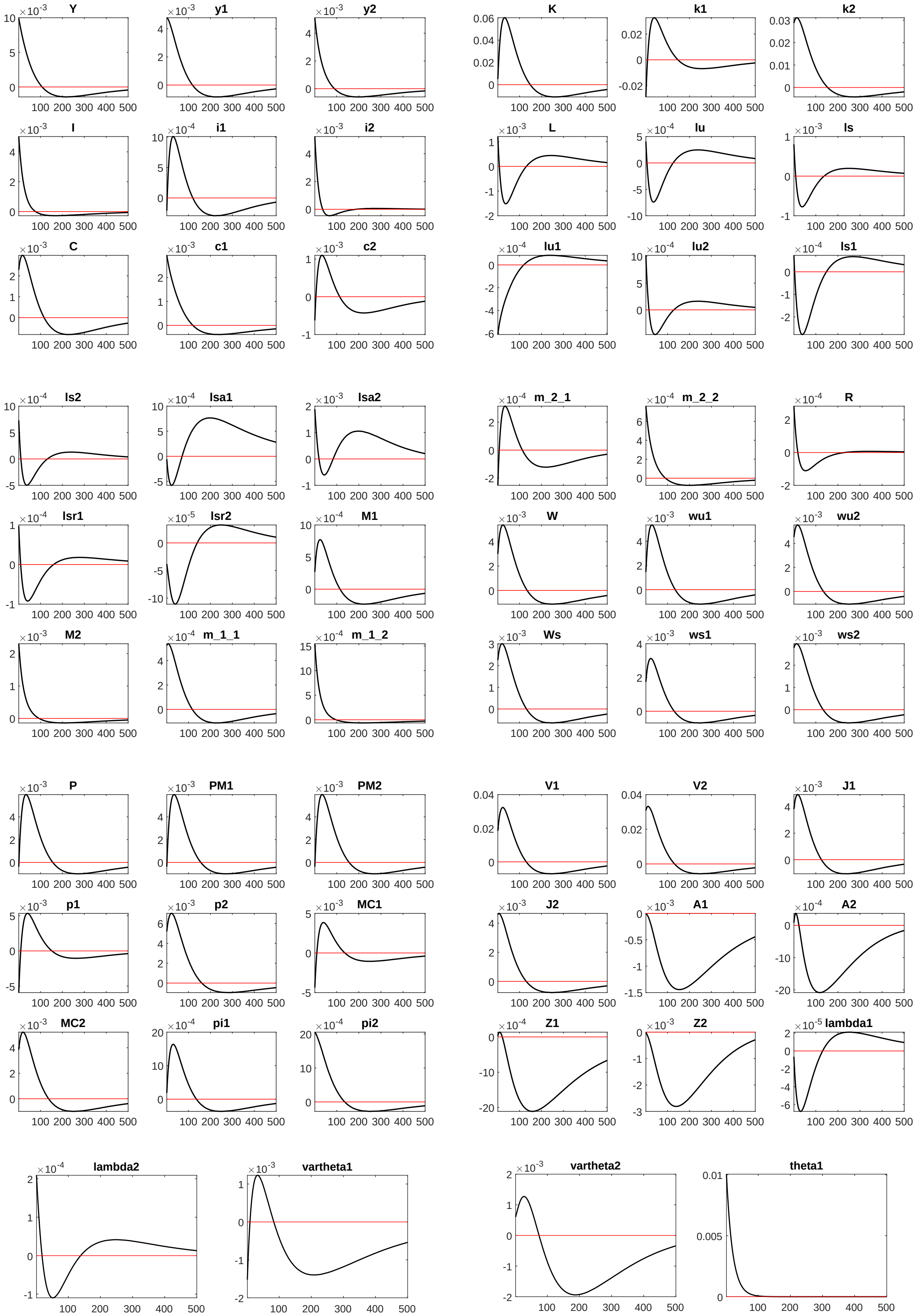


Figure 9: Impulse Response Functions Following 0.1 sd Productivity Shock (θ_1)



6 Calibration and Evaluation Against SVAR

After having succesfully built and simulated the model, I end by calibrating the multi-sector RBC and the full model for the US and comparing the results to that of the Structural Vector-Autoregression (SVAR). As mentioned in the introduction, I resort to calibration because I ran into presently insurmountable stochastic singularity or under-identification problems doing Bayesian estimation with the data currently at my disposal[10].

I will start this exercise by calibrating the models to the US economy divided into 3-sectors: Agriculture (AGR), industry (IND) and services (SER). I will also start calibrating the multi-sector RBC model to gain a benchmark against which the calibration of the full model can be evaluated.

6.1 US 3-Sector RBC vs. SVAR

Starting off with the basic 3-Sector RBC, I calibrate consumption (ω_i) and labor (ς_i) shares using the 10-Sector data provided by the Groningen Growth and Development Centre (GGDC) (M. Timmer et al., 2015), which I aggregate to 3-sectors and take the average sectoral shares over the period 1950-2012. The column-normalized input-output (IO) matrix is obtained by aggregating the World-Input-Output Database by M. P. Timmer et al. (2015) to 3-sectors and considering only the US domestic IO matrix, averaged over the available years 2000-2014.

Table 7: US Matrix of Input Shares (Columns), 2000-2014 Average

	AGR	IND	SER
AGR	0.242	0.019	0.009
IND	0.714	0.599	0.198
SER	0.043	0.383	0.793

Regarding the elasticities in the model, Herrendorf et al. (2013) estimate ϵ for the US 3-sector economy by considering long-run changes in broad sectors relative prices and final consumption expenditure shares, and come up with a benchmark estimate of 0.9. An extensive literature has estimated the Frisch labor supply elasticity and come up with values between 0.5 and 3, so I will keep $\varphi = 1.5$ (Chetty et al., 2011). I will also keep the stylized values of $\sigma = 2$, $\beta = 0.985$ and $\delta = 0.025$, which are broadly in line with the values estimated in the literature. Atalay (2017) estimates the elasticities of substitution among intermediate inputs η_i for 30 US sectors, and consistently finds values less than 0.2. Since Inputs from 3 broad sectors should be less substitutable than in a 30-sector setup, I will set all η_i to 0.1. Lastly, there is the elasticity of substitution ν among sectoral labor stocks. Horvath (2000) estimates an elasticity of 1 here.

Table 8: USA 3-Sector Parameterization

Parameter	Value	Parameter	Value
σ	2	φ	1.5
β	0.985	δ	0.025
ϵ	0.9	ν	1
ω_{AGR}	0.03	ς_{AGR}	0.03
ω_{IND}	0.28	ς_{IND}	0.25
ω_{SER}	0.69	ς_{SER}	0.71
α_{AGR}	0.3	β_{AGR}	0.3
α_{IND}	0.18	β_{IND}	0.2
α_{SER}	0.08	β_{SER}	0.66
η_{AGR}	0.1	ρ_{AGR}	0.95
η_{IND}	0.1	ρ_{IND}	0.95
η_{SER}	0.1	ρ_{SER}	0.95

[10]The stochastic singularity problems I encountered were not cause by some perfect linear relationship of observed variables in my equations - I have tried introducing additional shocks and measurement errors to prevent that - but, it seems, by the very nature of the model and the data.

With these parameters in place, I simulate a log-linear 1st-order approximation of the model over 100,000 periods, with equal-sized idiosyncratic shocks of standard deviation 0.1 to all sectors. The autocorrelations of and shock decompositions of Value-Added (VA) are shown in Table (9). The table shows that the presistence of the simulated series is roughly similar to that of the actual ones, taken from the GGDC.

Table 9: USA 3-Sector Autocorrelation Functions of Simulated and Observed Variables

Variable	1	2	3	4	5
y_{AGR}^{simul}	0.9311	0.8673	0.8078	0.7518	0.6994
y_{IND}^{simul}	0.9435	0.8904	0.8401	0.7926	0.7477
y_{SER}^{simul}	0.9468	0.8966	0.8488	0.8032	0.7597
y_{AGR}^{obs}	0.930	0.888	0.832	0.776	0.730
y_{IND}^{obs}	0.955	0.913	0.861	0.806	0.751
y_{SER}^{obs}	0.934	0.883	0.827	0.769	0.714

Table (10) below shows the simulated variance decomposition. It shows that agricultural productivity shocks only impact agriculture, while agriculture itself is equally impacted by shocks to Industry and to a lesser extent service sector shocks. Industrial output therewhile is driven to 92% by itself, and the service sector is driven to 68% by itself, while also responding with a 30% share to industrial shocks.

Table 10: USA 3-Sector Variance Decomposition Simulating one Shock at a Time (in %)

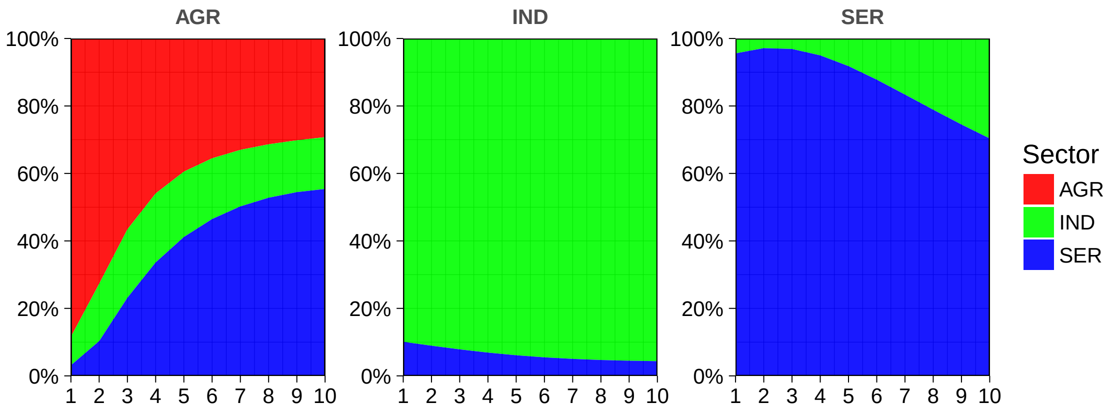
Variable	e_{AGR}	e_{IND}	e_{SER}	Total
y_{AGR}	37.65	37.05	24.79	99.49
y_{IND}	0.03	92.43	6.95	99.41
y_{SER}	0.53	30.60	67.69	98.82

This result from the model can be compared to the results of a Structural VAR (SVAR) estimated on the 3-sector VA data in log-levels. Diagnostic criteria such as the Schwartz-BIC suggest that in a VAR including a time-trend and a constant, 1 lag is enough to give serially uncorrelated errors. I also run the SVAR in growth rates, but the results are so similar that I only report the log-level SVAR. The SVAR is of the form

$$\mathbf{A}\mathbf{y}_t = \mathbf{C}(1)\mathbf{y}_{t-1} + \mathbf{B}\epsilon_t, \tag{192}$$

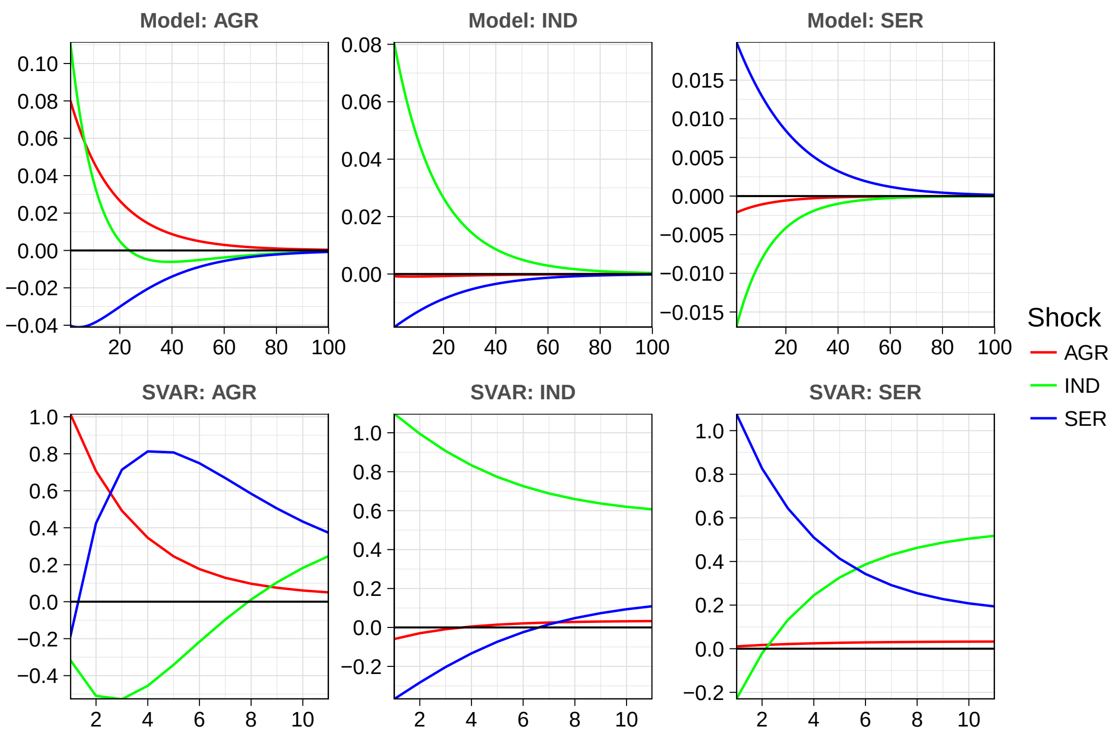
where $\mathbf{B} = \mathbf{I}_3$ is assumed diagonal, and the contemporaneous impact matrix $\mathbf{A}$ is replaced by the IO matrix shown in Table (7) but this time row-normalized (i.e. output shares are taken as more closely associated with the impact coefficients than input shares) and with 1's along the diagonal. The Forecast Error Variance Decomposition (FEVD) from this SVAR is shown in Figure (10). The FEVD after 10-periods is very closely in line with Table (10) except for agriculture, where services make up 55% of the variance and industry only 15% instead of 37% industry and 15% services as suggested by Table (10).

Figure 10: Forecast Error Variance Decomposition from SVAR



A final point of comparison are the Impulse-Response Functions (IRF's) from the model and the SVAR, shown in Figure (11). I note that both are on different time-scales (the SVAR has 10-period IRF's whereas the model reports 100-periods), and the shocks have different magnitudes (in the model the shock magnitude is 0.1 sd compared to 1 sd in the SVAR). Apart from these differences, the IRF's for industry and services are broadly similar (except for the response of SVAR: SER to IND which is non-stationary, but this effect disappears if the SVAR is run using growth rates of output instead of log-levels with trend, i.e. with growth rates the responses of SVAR: SER look very much like the responses of Model: SER). For agriculture however the IRF's differ between the model and the SVAR: In the model, agriculture responds positively to industry and negatively to services while in the SVAR this pattern is reversed. This pattern is the same whether the SVAR is run in log-levels + trend or in growth rates.

Figure 11: Impulse Response Function, Model vs. SVAR



Overall one can however conclude that the calibrated multi-sector RBC model already does a decent job in accounting for US broad-sectoral persistence and co-movement.

6.2 US 3-Sector RBC with Endogenous Technology

I proceed by calibrating the full model to the 3-sector US set-up. Apart from the parameters in the RBC, in the full model labor needs to be divided between skilled labor for R&D and adoption and unskilled labor. The World Bank Development indicators report that on average 3.8% of the American population work in R&D. Since the labor force only makes up around 60% of the population, and there is also labor devoted to technology adoption about which there is no information, I will assume that 2% of the US labor force is in R&D and adoption, thus $\varsigma_u = 0.98$ and $\varsigma_s = 0.02$. Unskilled labor is again divided over the three sectors using the same shares as in the RBC. Skilled labor must also be divided over the tree sectors, and here there is also unfortunately little information available[11]. It is clear that with the little share of agriculture in overall employment there is also little skilled labor devoted to R&D and adoption in agriculture. But since Agriculture in the US is very technology intensive, I will assume that the same share of 3% in skilled labor as in unskilled labor goes to Agriculture. For industry and services the weights however need to be changed, as most R&D is in industry, and I also expect the industry to be quicker in adopting new technologies. I will assume that 20% of the skilled labor is in services, and the remaining 77% in industry. For unskilled labor, I will assume the same elasticity of substitution $\nu_u = 1$ as in the RBC. Skilled labor however is much less substitutable, because it is more specialized. Therefore I set $\nu_s = 0.1$. For the labor share in adoption, Anzoategui et al. (2017) estimate an adoption elasticity of $\rho_a = 0.927$ for the entire US economy. I will set the same adoption elasticity in all sectors because not much is known about their technology adoption behavior. The same holds for R&D, Anzoategui et al. (2017) estimate $\rho_z = 0.376$ for the elasticity of new technologies w.r.t. R&D, which I also apply to all 3 sectors. Initially I will disable spillovers, i.e. $\rho_{Ma_i} = \rho_{Mr_i} = 0 \; \forall i$. Table (11) shows the autocorrelated functions of the simulated series. I include skilled and unskilled labor in the simulations because the IRF's show interesting patterns which I will discuss below. The simulated output series in Table (11) are a bit more persistent than the actual series shown in Table (9). The same holds true for the observed employment series (not reported), where the autocorrelation at lag 5 is typically down to 0.6.

Table 11: Autocorrelation Functions of Simulated Variables - Full Model

Variable	1	2	3	4	5
y_{AGR}	0.955	0.912	0.870	0.831	0.793
y_{IND}	0.962	0.926	0.891	0.857	0.824
y_{SER}	0.957	0.916	0.876	0.838	0.802
l_{uAGR}	0.959	0.919	0.881	0.845	0.810
l_{uIND}	0.956	0.914	0.872	0.833	0.795
l_{uSER}	0.980	0.960	0.942	0.924	0.907
l_{sAGR}	0.951	0.904	0.858	0.815	0.773
l_{sIND}	0.946	0.894	0.845	0.798	0.753
l_{sSER}	0.944	0.891	0.840	0.792	0.746

Table (12) shows the variance decomposition from all shocks. It shows that the R&D shocks in industry and services make up must of the variance, which indicates that a R&D shock of 0.1 sd is probably too large. So I reduce The shock magnitude of the R&D shock to 0.01 sd, and report the variance decomposition again in Table (13). Table (13) suggests that 70% of the variance in US output, skilled and unskilled labor is accounted for by R&D and productivity shocks in industry, and by skilled labor supply shocks. But again of course I have not estimated the variances of these shocks, so Tables (12) and Table (13) can at best give hints about the relative importance of different shocks - under the assumptions that the model is correctly specified and parameterized.

[11]The OECD publishes sectoral R&D statistics, but the sectors to not correspond or can easily be mapped to the three broad sectors considered here

Table 12: Variance Decomposition Simulating one Shock at a Time (in %) following 0.1 sd shocks - Full Model

Variable	e_{μ_u}	e_{μ_s}	$e_{\theta AGR}$	$e_{\theta IND}$	$e_{\theta SER}$	$e_{\chi AGR}$	$e_{\chi IND}$	$e_{\chi SER}$	Total
y_{AGR}	0.11	0.73	0.19	0.60	0.01	0.03	67.90	27.27	96.85
y_{IND}	0.11	0.72	0.19	0.63	0.01	0.03	67.63	27.31	96.65
y_{SER}	0.11	0.71	0.18	0.61	0.02	0.03	67.56	27.34	96.56
l_{uAGR}	0.11	0.73	0.19	0.61	0.01	0.03	67.74	27.30	96.72
l_{uIND}	0.11	0.73	0.19	0.61	0.01	0.03	67.90	27.27	96.86
l_{uSER}	0.10	0.66	0.17	0.64	0.01	0.03	66.53	27.50	95.66
l_{sAGR}	0.11	0.72	0.19	0.59	0.01	0.03	68.00	27.26	96.92
l_{sIND}	0.11	0.71	0.19	0.61	0.01	0.03	68.07	27.23	96.95
l_{sSER}	0.11	0.70	0.19	0.61	0.02	0.03	68.07	27.23	96.94

Table 13: Variance Decomposition Simulating one Shock at a Time (in %) following 0.1 sd shocks to μ and θ and 0.01 sd shocks to χ - Full Model

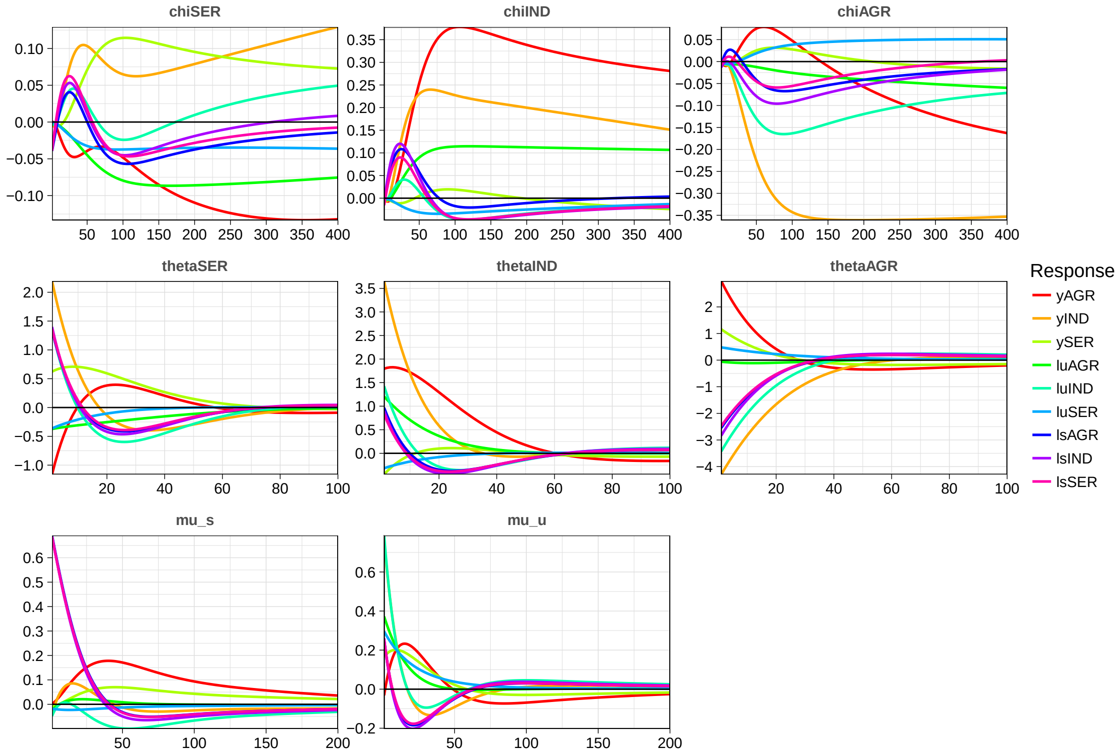
Variable	e_{μ_u}	e_{μ_s}	$e_{\theta AGR}$	$e_{\theta IND}$	$e_{\theta SER}$	$e_{\chi AGR}$	$e_{\chi IND}$	$e_{\chi SER}$	Total
y_{AGR}	4.08	26.82	6.96	21.96	0.41	0.01	24.87	9.99	95.11
y_{IND}	4.09	26.11	6.77	22.84	0.42	0.01	24.43	9.86	94.54
y_{SER}	3.89	25.93	6.74	22.32	0.77	0.01	24.80	10.04	94.49
l_{uAGR}	4.04	26.47	6.88	22.27	0.38	0.01	24.69	9.95	94.70
l_{uIND}	4.12	26.72	6.93	22.23	0.47	0.01	24.76	9.95	95.19
l_{uSER}	3.67	24.03	6.25	23.38	0.37	0.01	24.18	9.99	91.88
l_{sAGR}	4.07	26.74	6.91	21.98	0.46	0.01	25.13	10.07	95.39
l_{sIND}	4.01	26.25	7.00	22.46	0.54	0.01	25.23	10.09	95.58
l_{sSER}	3.99	26.00	6.99	22.54	0.56	0.01	25.35	10.14	95.58

In Figure (12) I show the IRF's from passing 0.1 sd shocks to all exogenous variables[12]. As is evident from the figure, the shocks differ in terms of magnitude and duration of the effect they produce. Productivity shocks (θ_i) have a large immediate impact, but only last for 50-80 periods, while R&D shocks (χ_i) have no immediate impact, develop their maximum impact after 100 periods, and last for more than 400 periods i.e. they have permanent growth effects. Shocks to the aggregate skilled and unskilled labor stocks (μ_u and μ_s) are in-between R&D and productivity shocks in terms of both magnitude and duration, with skilled labor supply shocks having more permanent effects than unskilled labor supply shocks.

[12]Note that Figure (12) is different from Figure (11) in that the shocks now make up the panels and the response variables are coloured. This was done because the effects of different shocks are different in magnitudes.

Figure 12: Impulse Response Functions to 0.1 sd Shocks ($\times 100$) - Full Model

(IRF's from 1st order log-linear approximation, model simulated over 100,000 periods)

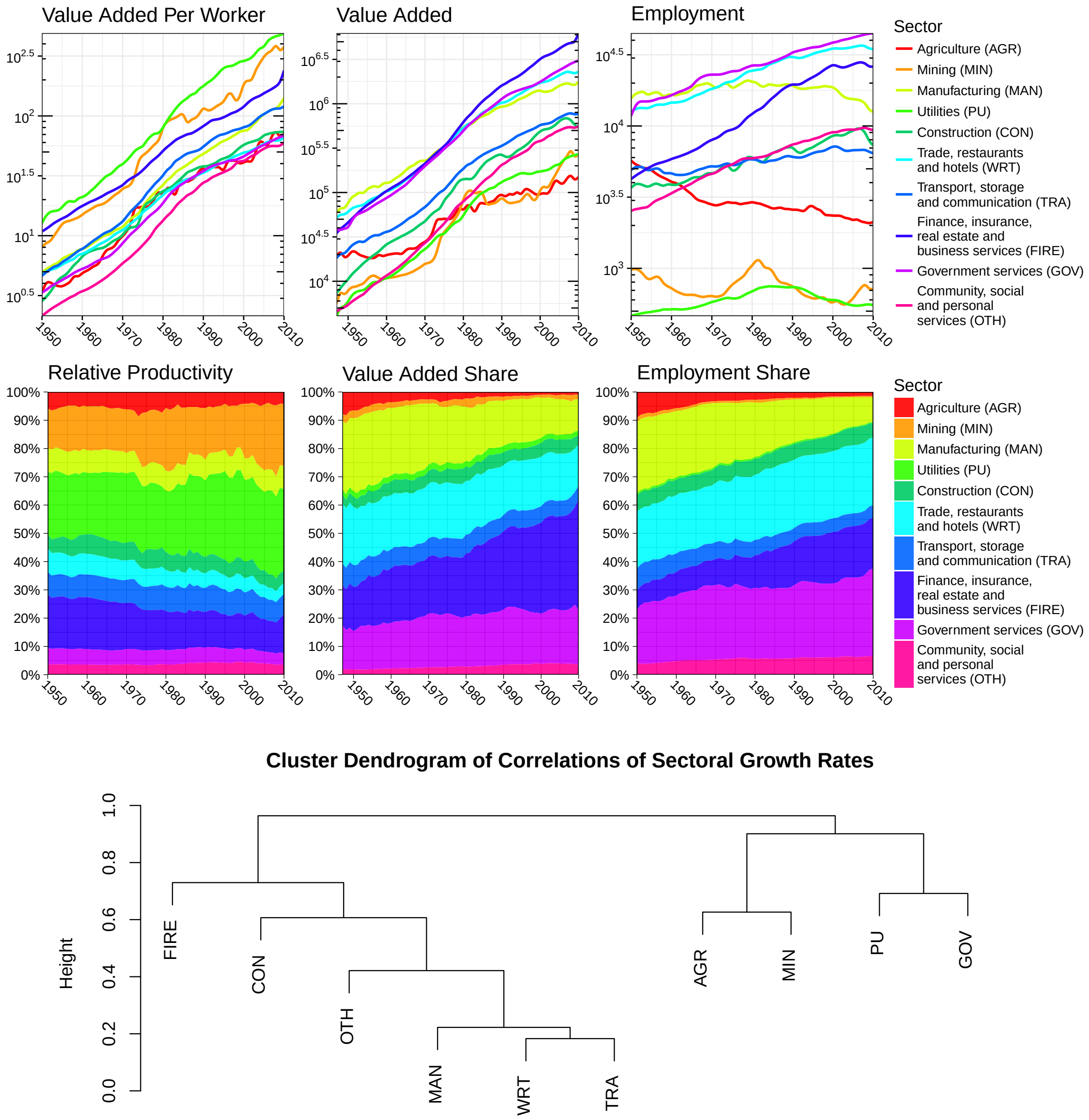


Regarding the direction of shock impacts, productivity shocks are broadly in line with R&D shocks. Agricultural productivity of R&D shocks increase agricultural output and services output, but decrease industrial output. This is broadly in-line with the SVAR. Industrial shocks increase industrial and agricultural output, but slightly decrease output in services. This is congruent to the RBC model but not to the SVAR, where industrial shocks also decrease agricultural output. Service sector shocks therewhile increase services and industrial output, but decrease agricultural output. This is not in-line with the RBC and SVAR results where service-sector shocks decrease industrial output. Concerning labor, skilled labor stocks in all sectors always co-move. Industry and service sector shocks increase the skilled labor stocks, but agricultural shocks decrease it. The response of unkilled labor is more heterogenous as stocks initially tend to co-move with outputs, while later the response might reverse. The model thus suggests that R&D shocks can have structural change effects, although the direction and magnitude of the change is not always reasonabl.

6.3 US 10-Sector RBC vs. SVAR

In subsection 6.1, I established that the multi-sector RBC model did not fare too bad in accounting for the impulse responses and variance decompositions of the three sectors, assuming of course that the SVAR is correctly identified. Now I want to take the model-evaluation a step further with the full 10-sector GGDC data of the US economy. The data is summarized in Figure (13). It shows only a slight pattern of structural change driven by a decline in agriculture, mining and manufacturing and an expansion of finance and the government. The dendrogram below shows the correlation structure of the 10 series - agriculture and mining and utilities and government co-move, and the remaining sectors also co-move with one-another: Manufacturing, trade and transport are closely correlated and community services, construction and finance covary to a lesser degree.

Figure 13: GGDC Data, United States 1950-2012



The corresponding 10-sector IO matrix is computed by aggregating the WIOD of M. P. Timmer et al. (2015) and shown (with normalized columns) in Table (14). It is only available for the years 2000-2014, and the average change in input-shares over this period is 0.025, with an average autocorrelation of 0.7 and and average time-correlation of 0.6. This gives hope that changes in the IO matrix at the 10-sector-level, on which the identification of the SVAR depends, are driven by the slow pattern of structural change observed in the data, and might not have completely changed since 1950.

Table 14: USA 2014 Column-Normalized Input-Output (IO) Matrix

	AGR	MIN	MAN	PU	CON	WRT	TRA	FIRE	GOV	OTH
AGR	0.370	0.001	0.078	0	0.003	0.006	0	0.001	0.001	0.002
MIN	0.008	0.211	0.105	0.173	0.017	0.001	0.005	0.002	0.008	0.003
MAN	0.299	0.237	0.469	0.218	0.475	0.162	0.246	0.071	0.226	0.171
PU	0.016	0.030	0.021	0.063	0.008	0.018	0.017	0.024	0.031	0.038
CON	0.011	0.031	0.005	0.029	0	0.005	0.008	0.038	0.035	0.011
WRT	0.112	0.059	0.101	0.077	0.279	0.100	0.082	0.065	0.083	0.087
TRA	0.058	0.082	0.051	0.130	0.052	0.120	0.244	0.058	0.065	0.055
FIRE	0.104	0.328	0.152	0.265	0.146	0.520	0.334	0.685	0.461	0.520
GOV	0.017	0.019	0.014	0.034	0.012	0.047	0.038	0.035	0.066	0.046
OTH	0.004	0.004	0.004	0.012	0.008	0.021	0.026	0.022	0.023	0.067

I parameterize the model with the average value added (ω_i) and employment (ς_i) shares, and the column-normalized IO matrix (γ_{ji}). All other parameters are as in the 3-sector set-up. Now by virtue of the larger disaggregation, providing an IO matrix with more zero or near-zero entries, a more sophisticated identification strategy can be followed for the SVAR. Again I will only run the VAR with log value added (VA) data since adding employment would make identification much more difficult and there are also only 60 years of joint data to estimate on. With 64 years of VA data and 10 variables, I experiment with lag lengths 1-4, including a constant and a linear trend in each equation. Among the information criteria, the Schwartz BIC suggest 1 as the optimal lag length, while AIC, FPE and HQIC suggest 4 as the optimal lag length. Diagnostic tests show that a VAR with one lag already produces serially incorrelated errors, jointly and on every series. Therefore to preserve degrees of freedom I adopt a VAR with one lag as my preferred specification, but I also report a VAR in growth rates of VA where all information criteria suggest an optimal lag-length of 1. The structural model again is:

$$\mathbf{A}\mathbf{y}_t = \mathbf{C}(1)\mathbf{y}_{t-1} + \mathbf{B}\epsilon_t. \tag{193}$$

To reach identification I assume that $\mathbf{B}$ is a 10×10 identity matrix. This means that $10(10-1)/2 = 45$ restrictions need to be imposed on $\mathbf{A}$. Since $\mathbf{A}$ captures the contemporaneous relationships between sectoral VA, I will impose restrictions on $\mathbf{A}$ by placing a number of zeros informed by the input-output matrix. Now a problem is that the IO matrix represents a dynamic system of production in which the rows describe the product-sale destinations (outputs) and the columns describe the gross value-added expenses (inputs) for each sector, while the matrix $\mathbf{A}$ contains in each row the contemporaneous impact coefficients of a shock to all other sectors on that sector. In the IO matrix this shock can come from two different directions, it could be a product-sale (demand) shock or an expenditure (supply) shock. To be able to place appropriate zeros, I take both channels into account by creating a potential impact matrix $\mathbf{A}^*$, computed as the output shares (normalized rows), to which I add the corresponding input shares (normalized columns), so that in each row i of $\mathbf{A}^*$, each entry j corresponds to the sum of the input and output shares of sector (column) j in sector (row) i's production[13].

$\mathbf{A}^* =$

	AGR	MIN	MAN	PU	CON	WRT	TRA	FIRE	GOV	OTH
AGR	1	0.008	0.905	0.014	0.013	0.124	0.053	0.099	0.022	0.005
MIN	0.005	1	0.900	0.080	0.041	0.050	0.076	0.282	0.047	0.005
MAN	0.084	0.097	1	0.026	0.063	0.146	0.095	0.186	0.132	0.017
PU	0.013	0.164	0.414	1	0.037	0.153	0.156	0.481	0.229	0.046
CON	0.014	0.037	0.478	0.026	1	0.271	0.072	0.644	0.306	0.020
WRT	0.029	0.009	0.429	0.027	0.110	1	0.168	0.669	0.179	0.041
TRA	0.014	0.018	0.385	0.035	0.030	0.240	1	0.483	0.157	0.039
FIRE	0.005	0.013	0.162	0.031	0.049	0.206	0.106	1	0.206	0.050
GOV	0.011	0.015	0.316	0.040	0.045	0.233	0.130	0.683	1	0.052
OTH	0.006	0.006	0.228	0.044	0.028	0.230	0.150	0.822	0.260	1

(194)

$\mathbf{A}$ is then obtained from $\mathbf{A}^*$ by assuming that all coefficients in $\mathbf{A}^*$ smaller than 0.044 are 0, i.e. sectors j whose overall input-and output interaction with sector i amounts to less than 5% of sector

[13] The input and outpute shares added in $\mathbf{A}^*$ take into account the rest of the World, that is they do not sum to 1.

i's VA are assumed to have a negligible direct impact on sector i's production. This assumption, together with the 1's along the diagonal, gives exactly the 45 restrictions necessary:

$$\mathbf{A} = \begin{array}{c|cccccccccc} & \text{AGR} & \text{MIN} & \text{MAN} & \text{PU} & \text{CON} & \text{WRT} & \text{TRA} & \text{FIRE} & \text{GOV} & \text{OTH} \\ \hline \text{AGR} & 1 & 0 & X & 0 & 0 & X & X & X & 0 & 0 \\ \text{MIN} & 0 & 1 & X & X & 0 & X & X & X & X & 0 \\ \text{MAN} & X & X & 1 & 0 & X & X & X & X & X & 0 \\ \text{PU} & 0 & X & X & 1 & 0 & X & X & X & X & X \\ \text{CON} & 0 & 0 & X & 0 & 1 & X & X & X & X & 0 \\ \text{WRT} & 0 & 0 & X & 0 & X & 1 & X & X & X & 0 \\ \text{TRA} & 0 & 0 & X & 0 & 0 & X & 1 & X & X & 0 \\ \text{FIRE} & 0 & 0 & X & 0 & X & X & X & 1 & X & X \\ \text{GOV} & 0 & 0 & X & 0 & X & X & X & X & 1 & X \\ \text{OTH} & 0 & 0 & X & 0 & 0 & X & X & X & X & 1 \end{array} . \tag{195}$$

The estimated impact matrix is:

$$\hat{\mathbf{A}} = \begin{array}{c|cccccccccc} & \text{AGR} & \text{MIN} & \text{MAN} & \text{PU} & \text{CON} & \text{WRT} & \text{TRA} & \text{FIRE} & \text{GOV} & \text{OTH} \\ \hline \text{AGR} & 1 & 0 & 0.089 & 0 & 0 & 0.106 & 0.106 & 0.106 & 0 & 0 \\ \text{MIN} & 0 & 1 & 0.094 & 0.100 & 0 & 0.106 & 0.106 & 0.111 & 0.106 & 0 \\ \text{MAN} & 0.109 & 0.099 & 1 & 0 & 0.097 & 0.097 & 0.100 & 0.100 & 0.102 & 0 \\ \text{PU} & 0 & 0.098 & 0.106 & 1 & 0 & 0.107 & 0.098 & 0.106 & 0.107 & 0.107 \\ \text{CON} & 0 & 0 & 0.101 & 0 & 1 & 0.097 & 0.109 & 0.096 & 0.096 & 0 \\ \text{WRT} & 0 & 0 & 0.101 & 0 & 0.097 & 1 & 0.096 & 0.100 & 0.101 & 0 \\ \text{TRA} & 0 & 0 & 0.102 & 0 & 0 & 0.098 & 1 & 0.085 & 0.101 & 0 \\ \text{FIRE} & 0 & 0 & 0.098 & 0 & 0.098 & 0.098 & 0.096 & 1 & 0.101 & 0.098 \\ \text{GOV} & 0 & 0 & 0.099 & 0 & 0.100 & 0.099 & 0.084 & 0.101 & 1 & 0.099 \\ \text{OTH} & 0 & 0 & 0.101 & 0 & 0 & 0.105 & 0.107 & 0.079 & 0.099 & 1 \end{array} . \tag{196}$$

The estimated coefficients look surprizingly homogeneous, but it turns out that plugging in the row-normalized IO matrix instead of $\hat{\mathbf{A}}$ only slightly changes the IRF's and FEVD's in the first two periods (the 0's are key for identification). A problem with just using the IO matrix as in the 3-sector set-up would be that it is ad-hoc and does not take into account demand elasticities and elasticities of substitution between different productive inputs, which the estimation should uncover. Therefore I only report the results with $\hat{\mathbf{A}}$, noting that the results with the IO matrix (i.e. full calibration) are quite similar. Starting the comparison again with the autocorrelations or simulated and observed variables, Figure (15) shows that the model is able to generate series of about the same persistence as the log-levels of the data.

Table 15: Autocorrelation Functions of Simulated and Observed Variables

	Lag	AGR	MIN	MAN	PU	CON	WRT	TRA	FIRE	GOV	OTH
M	1	0.96	0.95	0.96	0.97	0.95	0.96	0.96	0.96	0.96	0.96
O	2	0.92	0.90	0.92	0.94	0.90	0.92	0.92	0.93	0.92	0.92
D	3	0.88	0.86	0.89	0.91	0.86	0.89	0.89	0.89	0.88	0.88
E	4	0.84	0.82	0.85	0.88	0.82	0.85	0.85	0.86	0.84	0.85
L	5	0.81	0.78	0.82	0.85	0.78	0.82	0.82	0.83	0.80	0.81
D	1	0.96	0.95	0.96	0.96	0.96	0.96	0.96	0.96	0.96	0.96
A	2	0.92	0.90	0.92	0.91	0.91	0.93	0.92	0.91	0.92	0.92
T	3	0.88	0.85	0.87	0.87	0.87	0.88	0.88	0.87	0.87	0.88
A	4	0.84	0.79	0.83	0.83	0.82	0.84	0.84	0.83	0.83	0.84
	5	0.80	0.74	0.79	0.79	0.77	0.80	0.80	0.78	0.78	0.80

With the 10-sector model I have also compared the correlations of simulated and observed variables, although this is not straightforward since the log-levels of the data are very highly correlated due to the common trend, while the stationary series generated by the model are less correlated. I have therefore done the comparison with the correlation matrix of the growth rates of the observed series and also with the correlation matrix of the HP-filtered log-levels of the observed series. A summary statistic for the similarity of empirical and simulated correlations was generated by transforming all 3 correlation matrices (from the model, growth rates and HP-cycles of the data) into vectors (omitting the diagonal) and computed the correlation between these vectors i.e. the correlation between the correlation coefficients coming from the model and the data. The result is shown in Table (16).

Table 16: Correlation of Correlations of Simulated and Observed Variables

	Model	Growth	HP-Cycle
Model	1	0.080	0.138
Growth	0.080	1	0.786
HP-Cycle	0.138	0.786	1

It is evident from Table (16) that the model is able to generate a small part of the correlations between the variables, but by no means all of the joint variation. This result is confirmed by comparing the IRF's from the model with those of the SVAR and of the SVAR in first-differences (i.e. growth rates since the data are in log-levels) shown in Figure (14).

Figure 14: IRF's from 10-Sector SVAR's (1 lag) in VA vs. Model

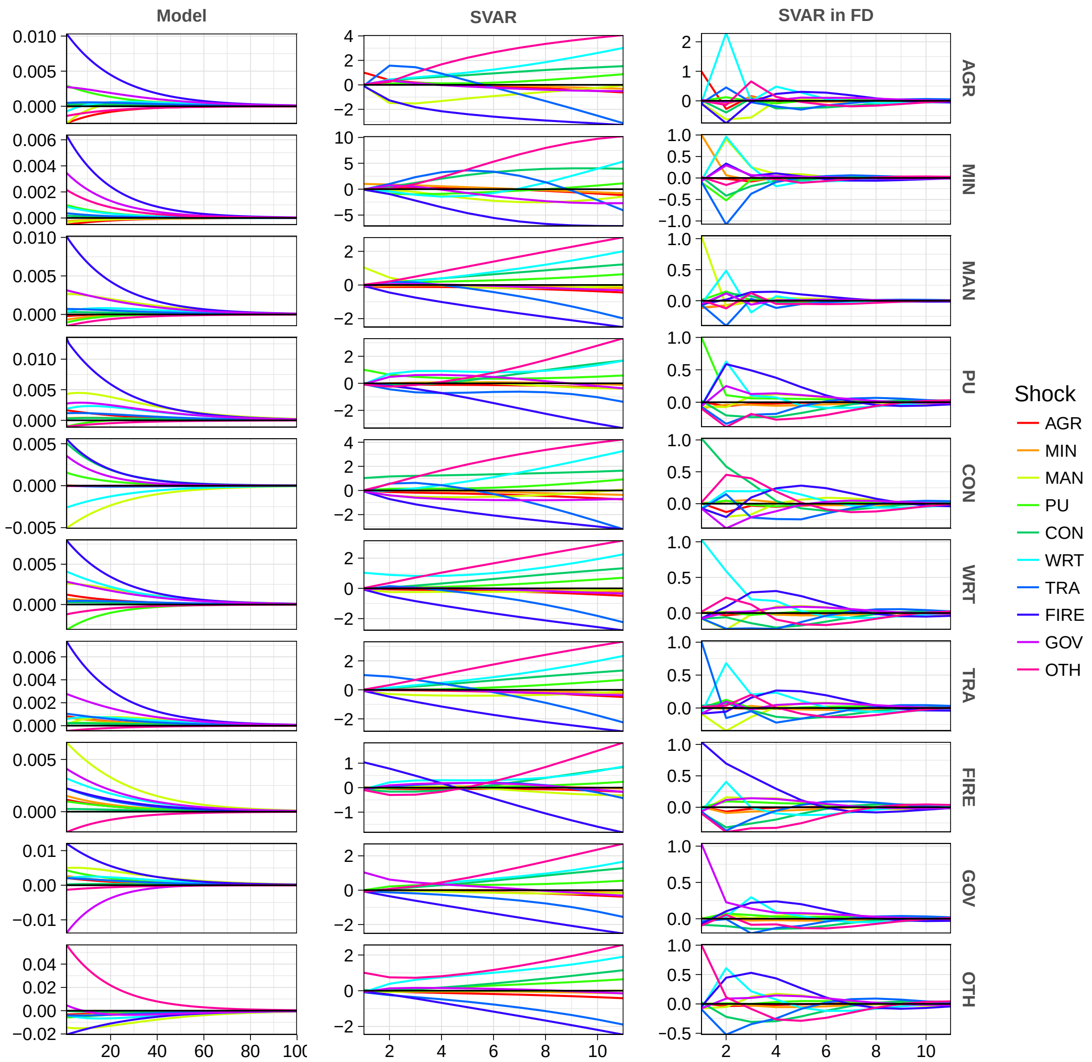


Figure (14) is hardly readable but shows that the IRF's coming from the model are quite different than the IRF's coming from the SVAR's. Again I summarize this comparison by calculating the cumulative impacts from each of the IRF's and correlating them, overall and by sector. Table

(17) shows the results of this exercise. In overall terms, there is no sizeable correlation between the cumulative impact multipliers coming from the model and from the two SVARS. The multipliers from the two SVAR's (SVARs) are also not positively correlated. This suggests that all the IRF's from the SVAR's are spurious or ill-identified. Disaggregating the impact multipliers by response-sector however shows some alignment of the model and the SVAR in FD among the responses of a number of sectors (FIRE, GOV, MAN, MIN, OTH and PU) to the different shocks. Disaggregating by shock-sector does not yield any alignment between the model and any of the the SVAR's, but the two SVAR's do produce similar responses to the same shocks.

Table 17: Correlation of Cumulative Shock Impacts from Model with SVAR's

Overall	SVAR : −0.06		SVAR in FD: 0.05		SVARs: −0.10	
	By Shock			By Response		
	SVAR	SVAR in FD	SVARs	SVAR	SVAR in FD	SVARs
AGR	0.02	-0.29	0.70	-0.70	0.02	0.23
CON	0.22	0.81	0.28	-0.47	-0.001	-0.35
FIRE	-0.15	-0.33	0.67	-0.69	0.56	-0.07
GOV	-0.24	-0.57	0.52	-0.68	0.62	-0.44
MAN	-0.02	-0.002	-0.12	-0.18	0.40	0.46
MIN	0.13	-0.03	0.67	-0.48	0.64	0.05
OTH	-0.07	0.30	0.44	-0.79	0.60	-0.34
PU	-0.07	-0.03	0.75	-0.41	0.49	-0.85
TRA	0.22	0.40	-0.33	-0.55	-0.15	-0.34
WRT	-0.22	0.11	0.75	0.63	-0.26	−0.16
Within	-0.08	0.02	0.34	-0.06	0.05	−0.10

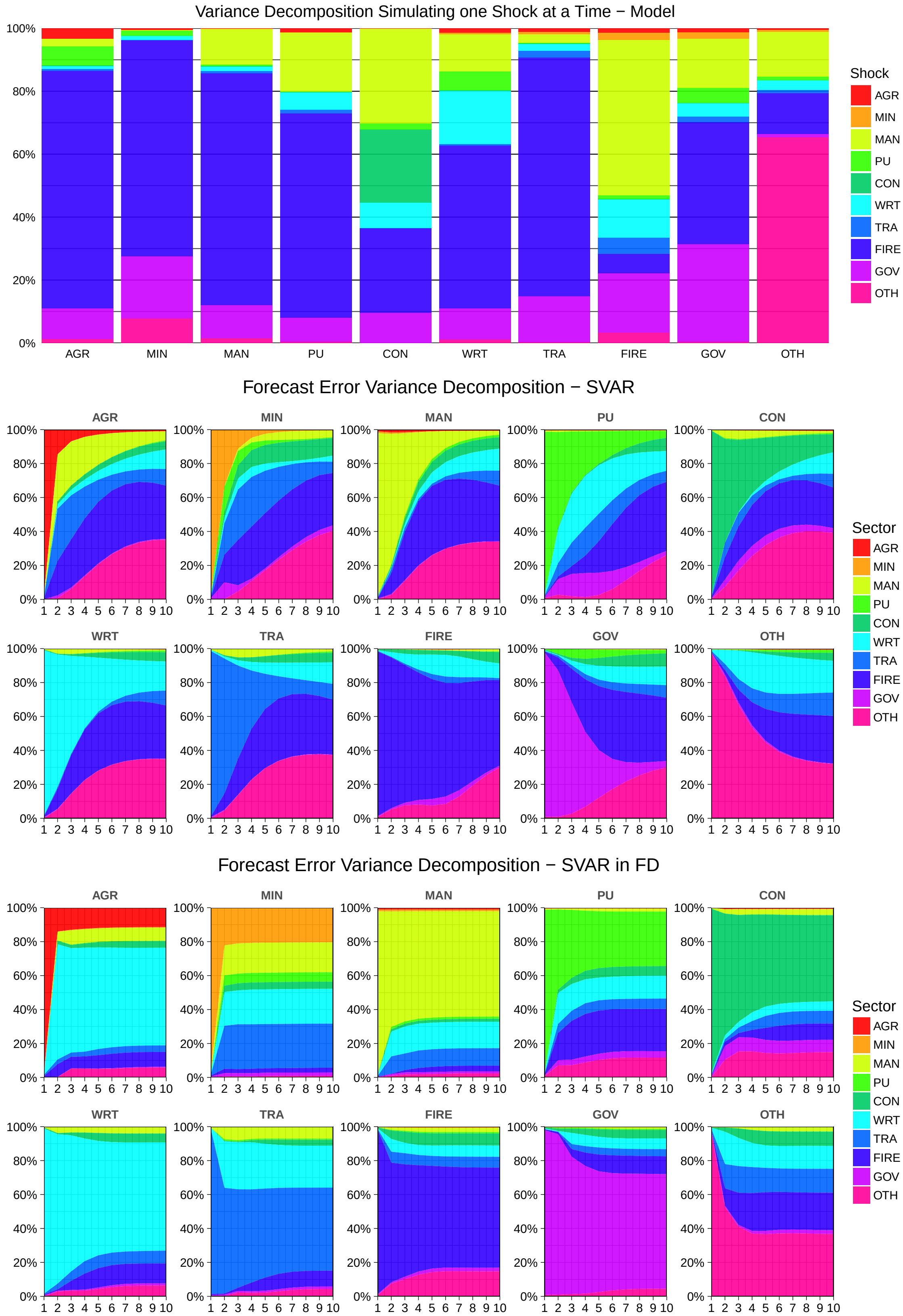
Up to this point it may be concluded that in a 10-sector setup the model is able to produce sectoral VA series that are similarly persistent as the observed series, but not much of the correlation structure between the variables. Before accepting the latter I will however also compare the variance decompositions, shown in Figure (15)[14]. Here the two SVAR's are much more aligned with one-another than in the IRF's, and also the model variance decomposition seems to have some similarity with that of the SVAR's - particularly for construction, government and other services. A curiosity is the large shares of finance and government in the variance decomposition of the model, which is much smaller in the SVAR's. Again I will break down the similarity of these variance decompositions to a number by correlating the shares. For the variance decompositions coming from the SVAR's, I consider both the average of the shares over the 10-periods, and the shares in the final (10th) period. The result is presented in Table (18), and indicates that there is some sizeable correlation between the variance shares of the different sectors coming from the model and the SVAR's, particularly the SVAR in log-levels. With a correlation of 0.7, the average FEVD's coming from the two SVAR's are also much more closely aligned.

Table 18: Correlation of Variance Decomposition Shares from Model and SVAR's

	Model	SVAR avg.	SVAR final.	SVAR in FD avg.	SVAR in FD final
Model	1	0.47	0.45	0.14	0.15
SVAR avg.	0.47	1	0.75	0.70	0.70
SVAR final	0.45	0.75	1	0.22	0.27
SVAR in FD avg.	0.14	0.70	0.22	1	0.99
SVAR in FD final	0.15	0.70	0.27	0.99	1

[14]To ease interpretation I have created a figure instead of a table from the variance decomposition of the model.

Figure 15: 10-Sector SVAR (1 lag) in VA, United States 1950-2012



This lets me conclude this exercise by suggesting that even if the multi-sector RBC model does not preform very well in this case, there is scope for improvement in matching the data via better calibration, i.e. by estimating also elasticities of substitution between the different inputs for all sectors, or via successful Bayesian estimation on better (monthly or quarterly) data. An important caveat in this evaluation was that the SVAR's did not seem well identified - as evidenced by the more-or-less spurious IRF's. The strong interdependence of productive sectors in the US, the dominance of some sectors such as finance as suppliers of inputs, and the different volatilities of the sectoral VA series evident from Figure (13) could all have contributed to identification problems in the SVAR.

7 Summary & Conclusion

I began this dissertation by reviewing facts about the business cycle and longer-term fluctuations in US economic activity, brought to light be two very relevant but so far distinct literatures in macroeconomics. The literature in sectoral shocks and aggregate fluctuations, starting with J. B. Long & Plosser (1983), has provided convincing evidence that up to 80% of US aggregate business-cycle fluctuations in output and productivity are generated by sectoral shocks cascading through the input-output network. The structure of the productive network is key in moderating the amplification of idiosyncratic sectoral shocks. Aggregate technology shocks, as found in many one-sector DSGE models, represent a significant abstraction with little resonance in the data. Meanwhile the literature on medium-run fluctuations and endogenous technological change, birthed with Comin & Gertler (2006), has challenged classical ways of viewing and modelling the business cycle and pointed out the more intricate and longer-lived nature of economic fluctuations. This literature has introduced the concept of the medium-run cycle comprising of all economic fluctuations above 200 quarters in period, and shown that this cycle can succesfully be accounted for - at least in the US - by DSGE models including endogenous mechanisms involving decentralized decisions to invest in R&D and technology adoption. The literature has further established that these investment and adoption decisions are taken pro-cyclically with the classical business cycle.

Against the backdrop of these recent developments in business-cycle macroeconomics, I motivated my project by suggesting the construction of a model that would be able to account for these extended fluctuations in output and productvity on the basis of decentralized sectoral decisions to invent and adopt new technologies, and sectoral interactions. I proceeded to construct such a model featuring endogenous R&D and technology adoption decisions in a multi-sector RBC economy, and introduced three levels of plausible interaction into it: (1) linkages in intermediate inputs, allowing for sectoral economic shocks to transmit through the input-output network influencing R&D and technology adoption decisions in other sectors, (2) spillovers in technology adoption, resulting from competitive pressures triggered by technology adoption by upstream or downstream sectors in the value chain, and (3) R&D spillovers, resulting from technological breakthroughs and increased R&D in either upstream or downstream sectors of the value chain. In the construction of the model I started off in section 3 with the simplest possible RBC model featuring heterogenous wages and prices and full interactions in intermediate productive inputs. A stylized calibration and simulation of this model showed that it is capable of delivering the expected dynamics in terms of relative wages, prices, consumption and intermediate inputs supplied following an indiosynchratic sectoral shock. Then, in section 4 I built the simplest possible RBC model encompassing the full endogenous R&D and technology adoption mechanism initially introduced by Comin & Gertler (2006) and enhanced in Anzoategui et al. (2017). A stylized calibration and simulation of this model showed that shocks to R&D and skilled labor supply have little immediate impact on the economy but produce extended increases in output, consumption, productivity, invented and adopted technology with gains from variety for the consumer, but also extended periods of higher unemployment. The impact of these shocks thus resembles a period of skill-biased technological change, which slowly phases-in and dies out even more slowly - just the behavior required to explain the long-waves in US output (coined the 'medium-term component' by Comin & Gertler (2006)).

Having built and analyzed separately the two key components of the model, I integrated in section 5 the two parts, yielding a complex multi-sector RBC economy with intermediate inputs in production, independent endogenous R&D and adoption decisions in each sector, and sectoral spillovers of R&D and technology adoption transmitted through the value chain. There remain some unsolved problems with this model, especially in terms of stability and dealing with non-stationarity, but a stylized calibration and simulation of the model showed that it in principle works in the intended way: sectoral R&D shocks transmit through the input-output network, increase sectoral outputs and trigger sectoral reallocations of productive investment, but also bring economic benefits to consumers in all sectors. Allowing for R&D and adoption spillovers magnifies the diffusion of technology, the increase in output, and the persistence of the shock, and generates additional benefits for consumers, but also additional unemployment. The simulation, however, also showed some additional caveats of the model in its current form, most notably in a frictionless set-up, R&D shocks to one sector produce such large reallocations and decline in capital and investment in that sector that the sector not experiencing the R&D shock actually reaps more of the economic benefits (in terms of investment and profits).

After bringing the model into existence and studying its basic properties via stylized calibration and simulation, it was my intention to perform a Bayesian estimation of the model on annual US 10-sectoral data in VA and employment since 1950, provided by (M. Timmer et al., 2015). Such an estimation and auxiliary procedures such as DSGE-VAR would have allowed me to evaluate the overall fit of the model to the US data and provided me with estimates of some parameters of the model that have not yet been estimated in the literature and that are very difficult to calibrate (particularly the parameters relating to sectoral R&D and technology adoption decisions and sectoral technology spillovers). My efforts to estimate the model have unfortunately drowned in insurmountable stochastic singularity problems, suggesting most likely the use of more detailed monthly or quarterly data and possibly some changes to the model structure as the way forward. In order to still present some sort of empirical exercise, I crudely calibrated the RBC and the full model to 3 US sectors - agriculture, industry and services, and the RBC also for 10-sectors, and compared the results with those from SVAR's estimated on the VA data of M. Timmer et al. (2015). The exercise comparing the RBC and the SVAR suggested that in a simple 3-sector set-up the simple calibrated RBC is roughly able to account for broad patterns of sectoral persistence and co-movement. In the 10-sector set-up the RBC is able to generate sectoral persistence but only little sectoral co-movement. However these results may also stongly be called into question by the evident idenification issues with the SVAR. The full model calibrated for 3 US sectors produced VA series that were a bit more persistent than the actual series, and showed that sectoral R&D shocks trigger extended increases in VA in broad US sector with a co-movement of skilled and unskilled labor. The IRF's were broadly in agreement with the response patterns in the simulation exercise of section 5, and, due to the persistence of the effect, additionally suggest that this model could also be interesting for understanding patterns of structural change.

If the efforts presented in this dissertation have established anything at this point, then I hope it is that they have provided some evidence for the relevance and potential of a disaggregated equilibrium model of production featuring endogenous R&D and technology adoption decisions - however inadequate it may be at this point - to account for broad patterns of extended real fluctuations at the aggregate and sectoral levels, at least in advanced economies similar to the US. There are many ways in which this line of research and the model constructed in this dissertation can be enhanced / extended. A first step would certainly be to get a Bayesian estimation of the model running on better - monthly or quarterly - sectoral data, and obtain estimates of the overall fir of the model to the data, and the importance of the various endogenous R&D and spillover channels - informed by the parameter estimates and standard errors. Afterwards, a next step would be to research how the fit of the model to the data can be improved. This could be achieved for example by means of introducing a limited number of frictions, such as investment adjustment costs, in key places, which might be capable of dramatically improving the models fit to the data. Potentially even the endogenous technology mechanism and the specification of the model itself may be adapted. The process of improving the model and its fit to the data could be informed by several steps. The first would be a detailed empirical investigation of sectoral data on VA, R&D and productivity which might stipulate alterations to the models specification. Second, the literature on multi-sector DSGE models and endogenous technology models may be studied more closely w.r.t. the details of estimation. The model of (Anzoategui et al., 2017) for example, which fits the aggregate US data remarkably well, incorporates many frictions (sticky wages, prices, investment adjustment costs and a few others). It would be useful to find out which of the frictions in Anzoategui et al. (2017) most improves the fit of their model and to consider implementing it in this model. Regarding the estimation of multi-sector DSGE model there are also many further references to consult, for example the guide by Dixon & Kara (2012). When the optimal specification of the model is found, the properties of the model in terms of accounting for aggregate and sectoral output patterns and patterns of structural change could also be investigated. Possibly the model specification could even be altered to allow for an alternative mechanism generating structural change as shown in Herrendorf et al. (2014), that is separate from R&D and technology decisions. Incorporating such a mechanism accounting for classical demand- and supply side factors driving structural change could dramatically improve the fit of the model when estimated on sectoral data. Thus, there remain many points of further improvement of the work presented here, and many new research avenues to be explored with this model.

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