Endogenous R&D and Technology Diffusion in a Multi-Sector RBC Economy

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Introduction

- This dissertation presents a multi-sector DSGE model combining a disaggregated account of production with an elaborate endogenous technological change mechanism.
- It is a Real-Business-Cycle (RBC) model. Unlike New-Keynesian DSGE models which use nominal frictions like Calvo-Pricing to generate persistent shocks, in this model persistence is achieved solely through sectoral interactions and endogenous technology responses.
- The model contributes to and synthesizes insights from two distinct literatures in macroeconomics:
    1. The literature on sectoral shocks and aggregate fluctuations
    2. The literature on medium-run cycles and endogenous technological change

Literature on Sectoral Shocks and Aggregate Fluctuations

- Models the economy with multiple sectors interacting through input-output linkages, focussing on the question to what degree sectoral shocks can generate, or are responsible for, aggregate business cycle volatility.

- Mostly RBC models that are carefully calibrated for 20-40 sectors in the US economy (2-digit ISIC level).

- Key contributions by Long & Plosser (1983), Horvath (1998, 2000), Petrella & Santoro (2011), Acemoglu et al. (2012, 2016), Bouakez et al. (2014), Stella (2015), and Atalay (2017)

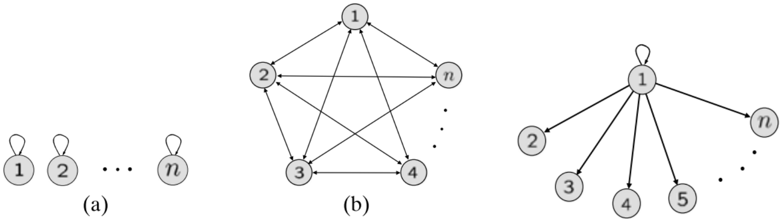
- Long & Plosser (1983): First Multisector RBC - Show that independent and serially uncorrelated shocks, lead to persistence and co-movement of sectoral outputs, and persistence of aggregate output (via consumer preferences, consumption smoothing and love for variety).

- Horvath (2000): Calibrated 36-sector model of US economy - Shows that limited interaction, characterized by a sparse IO matrix, reduces substitution possibilities among intermediate inputs which strengthens comovement in sectoral value-added.

→ Leads to a postponement of the law of large numbers which was hyothesized to cancel out the effects of various sectoral shocks on aggregate value-added (see e.g. Dupor (1999)).

- Acemoglu et al. (2012): Idiosyncratic sectoral shocks may lead to aggregate fluctuations, but rate at which aggregate volatility decays is determined by the structure of the network capturing such linkages.

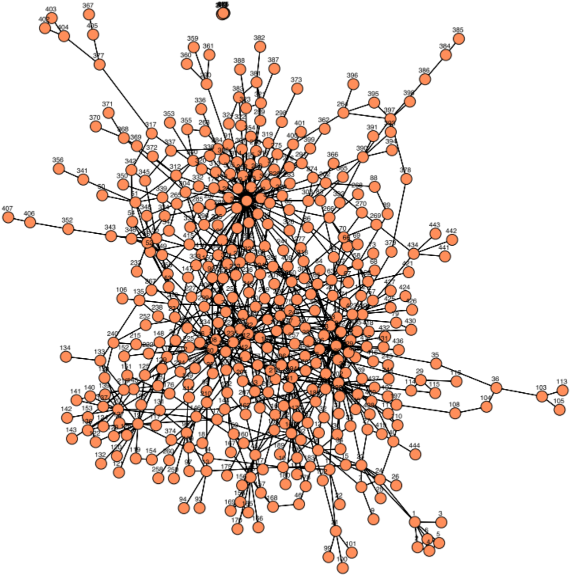
→ Sizeable aggregate volatility is only obtained if there exists significant asymmetry in the roles that sectors play as suppliers to others. The 'sparseness' of the IO matrix per se is unrelated to the nature of aggregate fluctuations.



- Atalay (2017): Quantifies the contribution of sectoral shocks to business cycle fluctuations in aggregate US output, using data on U.S. industries input prices and input choices.

→ Complementarities in inputs indicate that industry-specific shocks are substantially more important than previously thought, accounting for at least half of aggregate volatility (his estimate is 80% or aggregate volatility).

US Input-Output Network:



Literature on Medium Run Business Cycles

- Key contributions by Comin & Gertler (2006), Comin (2009), Bianchi et al. (2018) and Anzoategui et al. (2017).

- Comin & Gertler (2006): seminal work: define as the medium-term cycle the sum of the high- and medium-frequency variation in the data (frequencies $\leq$ 200 quarters).

$\rightarrow$ Substantially more volatile and persistent than conventional business-cycles (32 quarters, HP filter). Fluctuations exhibit significant procyclical movements in technological change, R&D, and efficiency and intensity of resource utilization.

$\rightarrow$ DSGE Model of the medium term cycle: endogenous strategic decisions by firms and other economic agents to invest in R&D and adopt new technologies happen pro-cyclical to the classical business cycle and introduce medium-run fluctuations.

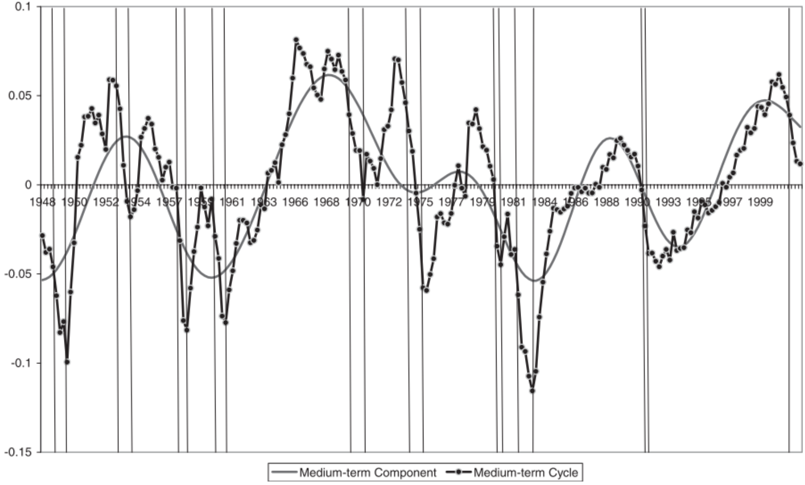


FIGURE 1. NONFARM BUSINESS OUTPUT PER PERSON 16–65

- Comin (2009): Presents evidence on the relevance of macro models where endogenous technological change mechanisms are responsible both for long-run growth and the propagation of low-persistence shocks.

→ Simple DSGE model of endogenous technological change and diffusion that is consistent with the evidence.

- Anzoategui et al. (2017): Stipulate that slowdown in productivity following the Great Recession (2008/09 crisis) was in significant part an endogenous response.

→ Present panel data evidence that technology diffusion is highly cyclical, develop and estimate a rich New-Keynesian DSGE model with endogenous R&D and technology adoption mechanism, and show that the model's implied cyclicality of technology diffusion is consistent with the panel data evidence.

This Model

- Production and innovation in each sector is decentralized involving 4 independent optimizing agents (following Anzoategui et al. (2017)):

  1. Perfectly competitive final goods (retail) firms
  2. Monopolistically competitive wholesale firms
  3. Technology adopters
  4. Technology innovators

- The latter two also reap benefits of imperfect competition in the wholesale sector by selling production plans and ideas

- Benefits from technology creation and adoption will be in terms of expanding variety = expansion of the number of wholesale firms, each producing a differentiated product

- A distinction between technology creation and adoption is made to allow for realistic lags in the adoption process
- Interaction between sectors is allowed to take place in 3 different ways:
    1. Intermediate input (and demand) linkages: Independent sectoral shocks can generate pro-cyclical responses in terms of R&D and technology adoption decisions in other sectors.
    2. Strategic complementarities inside productive value chains: Lead to 'spillovers' following the adoption of new technologies in upstream/downstream sectors (pressures to increase productive efficiency throughout the value chain).
    3. R&D spillovers also arising from IO interaction (e.g. increased R&D in electric cars may also increase R&D in battery technology and vice-versa).

RBC with Endogenous Technology

Final Good (Retail) Firms:

The final goods firm is perfectly competitive and aggregates intermediate goods produced by a continuum (measure a_t, where a_t is the stock of adopted technologies) of wholesale firms:

$$y_t = \left(\int_0^{a_t} y_{kt}^{\frac{\psi-1}{\psi}} dk \right)^{\frac{\psi}{\psi-1}} . \tag{1}$$

It maximizes revenues taking aggregate and input prices as given:

$$\max_{y_{kt}} \quad p_t \left(\int_0^{a_t} y_{kt}^{\frac{\psi-1}{\psi}} dk \right)^{\frac{\psi}{\psi-1}} - \int_0^{a_t} p_{kt} y_{kt} dk \tag{2}$$

Taking the FOC w.r.t. any particular y_{kt} yields the demand function for wholesale good k, which is directly proportional to aggregate demand and inversely proportional to its relative price:

$$y_{kt} = y_t \left(\frac{p_t}{p_{kt}} \right)^{\psi} . \tag{3}$$

Substituting the demand function back in the aggregator function yields the ideal price index:

$$p_t = \left(\int_0^{a_t} p_{kt}^{1-\psi} dk \right)^{\frac{1}{1-\psi}} . \tag{4}$$

Since in this model all wholesale firms are identical in their pricing behavior, Eq. (4) can be rewritten as:

$$p_t = a_t^{\frac{1}{1-\psi}} p_{kt} \quad \text{or} \quad p_{kt} = a_t^{\frac{1}{\psi-1}} p_t. \tag{5}$$

The same is true for output, Eq. (1) can be written as:

$$y_t = a_t^{\frac{\psi}{\psi-1}} y_{kt} \quad \text{or} \quad y_{kt} = a_t^{\frac{\psi}{1-\psi}} y_t. \tag{6}$$

Intermediate Goods (Wholesale) Firms:

The representative intermediate goods firm chooses capital k_{kt}, unskilled labor l_{ukt} to produce output by the following technology:

$$y_{kt} = \theta_t k_{kt}^{\alpha} l_{ukt}^{1-\alpha}. \tag{7}$$

θ_t is a stationary productivity shock to the intermediate goods sector. Firms then choose inputs and the price subject to the final good firms (consumers) demand function given by Eq. (3):

$$\max_{k_{kt},\ l_{ukt}} \quad \pi_{kt} = p_t y_t^{\frac{1}{\psi}} \left(\theta_t k_{kt}^{\alpha} l_{ukt}^{1-\alpha}\right)^{\frac{\psi-1}{\psi}} - r_t p_t k_{kt} - w_{ut} l_{ukt}. \tag{8}$$

Assuming each intermediate good firm is very small w.r.t. the whole of intermediate goods firms, so that it's choice of inputs does not impact the aggregate price or quantity, yields the FOC's:

$$\underbrace{p_{kt}\alpha\frac{y_{kt}}{k_{kt}}}_{\text{MR(k)}} = \frac{\psi}{\psi-1}\underbrace{r_t p_t}_{\text{MC(k)}}, \tag{9}$$

$$\underbrace{p_{kt}(1-\alpha)\frac{y_{kt}}{l_{ukt}}}_{\text{MR(l)}} = \frac{\psi}{\psi-1}\underbrace{w_{ut}}_{\text{MC(l)}}. \tag{10}$$

Inserting these FOC's back into the inverse demand function gives the optimal pricing choice of the individual wholesale firm:

$$p_{kt} = \frac{\psi}{\psi-1}\underbrace{\frac{1}{\theta_t}\left(\frac{r_t p_t}{\alpha}\right)^{\alpha}\left(\frac{w_{ut}}{1-\alpha}\right)^{1-\alpha}}_{\text{MC}}. \tag{11}$$

This is the standard Dixit & Stiglitz (1977) result that in a monopolistically competitive equilibrium the price is a constant mark-up over marginal cost.

Technology Adopters[1]:

Let z_t be the stock of invented technologies. The probability $0 < \lambda_t < 1$ that a new technology is adopted is given by

$$\lambda_t = \kappa(z_t l_{sat})^{\rho_a}, \tag{12}$$

where κ and $0 < \rho_a < 1$ are constants ($\lambda' > 0,\ \lambda'' < 0$), and l_{sat} is the skilled labor investment devoted to technology adoption in each period[2]. The value to the adopter of successfully bringing a new technology into use, v_t, is given by the present value of intermediate good firm profits from operating the technology

$$v_t = \pi_t + \phi E_t \frac{v_{t+1}}{1 + r_{t+1}}, \tag{13}$$

[1] Intermediate goods are first invented and then adopted, this describes their adoption conditional on their invention. The adoption process is procyclical but takes time. It is also decentralized e.g. aggregate patterns are modelled without taking account of individual firms adoptions. In each period a fraction of the available new technologies become usable. Whether a technology becomes usable is a random draw with success probably λ_t. Once a technology is usable, all firms are able to employ it immediately, which is modelled by an expansion in the number of varieties a_t (as the adopter sells the technology to a new intermediate goods firm (a start-up)). pro-cyclical adoption behavior is obtained by endogenizing the probability λ_t that a new technology becomes usable and making it increasing in the amount of resources devoted to adoption.

[2] The presence of z_t accounts for the fact that the adoption process becomes more efficient as the technological state of the economy improves.

where ϕ is the probability that the technology survives (i.e. does not become obsolete), which works like a discount factor here. Since adoption of a technology is stochastic with probability λ_t, the adopter chooses l_{sat} to maximize the value J_t gained from the acquisition of unadopted technologies:

$$\max_{l_{sat}} \quad J_t = \phi E_t \left\{ \frac{\lambda_t v_{t+1} + (1 - \lambda_t) J_{t+1}}{1 + r_{t+1}} \right\} - w_{st} l_{sat}. \tag{14}$$

The first term in the Bellman equation represents the discounted benefit from acquiring technologies: the probability weighted sum of the values of adopted and unadopted technologies. The FOC describing optimal skilled labor supply is:

$$w_{st} = z_t \lambda_t' \phi E_t \left\{ \frac{v_{t+1} - J_{t+1}}{1 + r_{t+1}} \right\} = \rho_a \frac{\lambda_t}{l_{sat}} \phi E_t \left\{ \frac{v_{t+1} - J_{t+1}}{1 + r_{t+1}} \right\}. \tag{15}$$

The FOC equates the marginal gain from adoption expenditures - the increase in λ_i times the discounted difference between the value of adopted versus unadopted technology - to the marginal cost w_{st}.

The term $v_{t+1} - J_{t+1}$ is pro-cyclical, by virtue of the greater influence of near term profits on the value of adopted technologies relative to unadopted ones. As a consequence, I_{sat} and pace of adoption λ_t also vary pro-cyclically[3].

Finally, the evolution of adopted technologies is:

$$a_{t+1} = \lambda_t \phi[z_t - a_t] + \phi a_t, \tag{16}$$

where $z_t - a_t$ is the stock of technologies available for adoption.

[3] Wage-stickiness may also be required to generate the full effect.

Technology Innovators:

Innovators use skilled labor to create new ideas, which adopters can buy and transform into production plans for intermediate goods bought by wholesale firms. Let ϑ_t be the marginal product of skilled labor producing a technology in a given time-period:

$$\vartheta_t = \chi_t z_t l_{srt}^{\rho_z - 1}, \tag{17}$$

where l_{srt} is skilled labor working on R&D. As in Romer (1990), the presence of z_t makes this a linear growth model. It is assumed that $\rho_z < 1$, implying that increased employment of skilled labor reduces its productivity for R&D. χ_t is an exogenous productivity shifter following a stochastic process:

$$\log \chi_t = (1 - \rho_\chi) \log \chi^* + \rho_\chi \log \chi_{t-1} + \epsilon_t^\chi. \tag{18}$$

The representative innovator chooses l_{srt} to maximize the expected value of the technology, as given by Eq. (14):

$$\underset{l_{srt}}{max} \quad E_t \frac{l_{srt} \vartheta_t J_{t+1}}{1 + r_{t+1}} - w_{st} l_{srt}. \tag{19}$$

The FOC equates the maginal discounted benefit of an additional unit of skilled labor in innovation with it's marginal cost:

$$E_t \frac{\vartheta_t J_{t+1}}{1 + r_{t+1}} = E_t \frac{\chi_t z_t l_{srt}^{\rho_z - 1} J_{t+1}}{1 + r_{t+1}} = w_{st}. \tag{20}$$

Given that profits from intermediate goods are pro-cyclical, the value of an unadopted technology, which depends on expected future profits, will also be pro-cyclical. Recall that ϕ is the survival rate for any given technology. Then, we can express the evolution of technologies as:

$$z_{t+1} = \phi z_t + \vartheta_t l_{srt} \quad \text{or} \quad \frac{z_{t+1}}{z_t} = \phi + \chi_t l_{srt}^{\rho_z}. \tag{21}$$

After aggregating the equations for intermediate goods firms, adopters and innovators, solving a consumers problem (CRRA) for consumption, skilled and unskilled labor supply, and adding an equilibrium condition and labor supply shocks, the RBC model with endogenous technology is given by the following equations:

Equation	Definition
$l_{ut}^{\varphi} = \varsigma_u \mu_u \frac{w_{ut}}{c_t^{\sigma} p_t}$	Unskilled Labor Supply
$l_{st}^{\varphi} = \varsigma_s \mu_s \frac{w_{st}}{c_t^{\sigma} p_t}$	Skilled Labor Supply
$c_t^{-\sigma} = \beta E_t \left[c_{t+1}^{-\sigma} (1 - \delta + r_{t+1}) \right]$	Euler Equation
$k_{t+1} = (1-\delta) k_t + i_t$	Capital Law of Motion
$y_t = a_t^{\frac{1}{\psi - 1}} \theta_t k_t^{\alpha} l_{ut}^{1-\alpha}$	Production Function
$a_t^{\frac{1}{1-\psi}} \alpha \frac{y_t}{k_t} MC_t = r_t p_t$	Demand for Capital
$a_t^{\frac{1}{1-\psi}} (1-\alpha) \frac{y_t}{l_{ut}} MC_t = w_{ut}$	Demand for Labor
$MC_t = \frac{1}{\theta_t} \left(\frac{r_t p_t}{\alpha} \right)^{\alpha} \left(\frac{w_{ut}}{1-\alpha} \right)^{1-\alpha}$	Marginal Cost
$a_t^{\frac{1}{\psi - 1}} p_t = \frac{\psi}{\psi - 1} MC_t$	(Optimal) Price Level
$\lambda_t = \kappa (z_t l_{sat})^{\rho_a}$	Adoption Success Probability
$\Pi_t = p_t a_t^{\frac{1}{\psi - 1}} \theta_t k_t^{\alpha} l_{ut}^{1-\alpha} - r_t p_t k_t - w_{ut} l_{ut}$	Intermediate Goods Aggregate Profit
$v_t^a = \Pi_t + \phi E_t \frac{v_{t+1}^a a_t}{a_{t+1}(1+r_{t+1})}$	Value of Adopted Technology
$J_t^z = E_t \left\{ \frac{\lambda_t v_{t+1}^a \frac{z_t}{a_{t+1}} + (1-\lambda_t) J_{t+1}^z \frac{z_t}{z_{t+1}}}{1+r_{t+1}} \right\} - w_{st} l_{sat} z_t$	Value of Unadopted Technology
$w_{st} l_{sat} = \rho_a \lambda_t \phi E_t \left\{ \frac{\frac{v_{t+1}^a}{a_{t+1}} - \frac{J_{t+1}^z}{z_{t+1}}}{1+r_{t+1}} \right\}$	Optimal Adoption Investment
$a_{t+1} = \lambda_t \phi [z_t - a_t] + \phi a_t$	Evolution of Adopted Technology
$\vartheta_t = \chi_t z_t l_{srt}^{\rho_z - 1}$	Productivity of R&D
$E_t \frac{\frac{\vartheta_t}{z_{t+1}} J_{t+1}^z}{1+r_{t+1}} = w_{st}$	Optimal R&D Investment
$z_{t+1} = \phi z_t + \vartheta_t l_{srt}$	Evolution of Technology
$l_{st} = (z_t - a_t) l_{sat} + l_{srt}$	Skilled labor Aggregation
$y_t = c_t + i_t$	Equilibrium Condition
$\log \chi_t = (1-\rho_\chi) \log \chi^* + \rho_\chi \log \chi_{t-1} + \epsilon_t^{\chi}$	R&D Shock
$\log \theta_t = \rho_\theta \log \theta_{t-1} + \epsilon_t^{\theta}$	Productivity Shock
$\log \mu_{ut} = \rho_{\mu_u} \log \mu_{u,t-1} + \epsilon_{ut}^{\mu}$	Unskilled Labor Supply Shock
$\log \mu_{st} = \rho_{\mu_s} \log \mu_{s,t-1} + \epsilon_{st}^{\mu}$	Skilled labor Supply Shock

Extension to N-Sectors

Now constructing an integrated N-sector RBC economy in which each sector has its own retailers, wholesale firms, technology adopters and technology innovators:

- Mostly similar equations, but indexed by i to denote the sector, and need to solve a few allocation problems regarding consumption bundles, skilled and unskilled labor supply to different sectors, and optimal choice of intermediate inputs.
- Additional terms are added to the endogenous technology equations to enable R&D and adoption spillovers between interlinked sectors.

Intermediate Goods (Wholesale) Firms:

The representative intermediate goods firm in sector i chooses capital k_{kit}, unskilled labor l_{ukit} and intermediate goods from other sectors (j) M_{kit} to produce output by the following Cobb-Douglas technology

$$y_{kit} = \theta_{it} k_{kit}^{\alpha_i} l_{ukit}^{\beta_i} M_{kit}^{1-\alpha_i-\beta_i} \quad \forall\, i, \tag{22}$$

with intermediate inputs composite:

$$M_{kit} = \left[\sum_{j=1}^{N} \gamma_{ji}^{\frac{1}{\eta_i}} m_{jkit}^{\frac{\eta_i - 1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i - 1}} \quad \forall\, i. \tag{23}$$

The notation is $m_{ji} = m_{\text{origin}\to\text{destiny}}$. θ_{it} is a stationary productivity shock to all wholesale firms in sector i.

Technology Adopters:

The sector-specific adoption success probability $0 < \lambda_{it} < 1$ is given by a concave function

$$\lambda_{it} = \kappa_i \left(\omega_{adi} \sum_{j=1}^{N} \gamma_{ji} a_{jt} + \omega_{aui} \sum_{j=1}^{N} \gamma_{ij} a_{jt} \right)^{\rho_{Mai}} (z_{it} l_{sait})^{\rho_{ai}} \quad \forall\, i, \tag{24}$$

where κ, $0 < \rho_{Ma} < 1$ and $0 < \rho_a < 1$ are constants $(\lambda' > 0,\ \lambda'' < 0)$. The first term reflects adoption learning spillovers from other sectors, where the first sum reflects adoption pressures resulting from upstream sectors in the value chain (i.e. sectors that supply inputs to sector i), and the second sum reflects adoption pressures from the downstream sectors (i.e. sectors that buy sector i's output). These spillovers reflect the input-output-mix in the wholesale sector, and their intensity is regulated by ρ_{Mai}, and the weights ω_{adi} and ω_{aui} reflecting the relative importance of downstream and upstream pressures.

Technology Innovators:

Let l_{srit} be skilled labor employed in R&D by the representative innovator in sector i and let ϑ_{it} be the marginal product of skilled labor producing a technology in a given time-period

$$\vartheta_{it} = \chi_{it} z_{it} \left(\omega_{rdi} \sum_{j \neq i} \gamma_{ji} z_{jt} + \omega_{rui} \sum_{j \neq i} \gamma_{ij} z_{jt} \right)^{\rho_{Mri}} l_{srit}^{\rho_{zi}-1} \quad \forall\, i. \quad (25)$$

Again $0 < \rho_{zi} < 1$, implying that increased R&D in the aggregate reduces the efficiency of R&D at the individual level. Also $\rho_{Mri} < 1$, so that there are diminishing returns to upstream or downstream innovation for the sector's own innovation process.

Households:

Aggregate consumption is a CES aggregate of consumption goods produced by N sectors, skilled labor l_{st} and unskilled labor l_{ut} are CES aggregates of sectoral skilled and unskilled labor stocks

$$c_t = \left[\sum_{i=1}^{N} \omega_i^{\frac{1}{\epsilon}} c_{it}^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}}, \qquad l_t = l_{ut} + l_{st}, \tag{26}$$

$$l_{ut} = \left[\sum_{i=1}^{N} \varsigma_{ui}^{\frac{1}{\nu_u}} l_{uit}^{\frac{\nu_u-1}{\nu_u}}\right]^{\frac{\nu_u}{\nu_u-1}}, \qquad l_{st} = \left[\sum_{i=1}^{N} \varsigma_{si}^{\frac{1}{\nu_s}} l_{sit}^{\frac{\nu_s-1}{\nu_s}}\right]^{\frac{\nu_s}{\nu_s-1}}. \tag{27}$$

Skilled labor in each sector is again divided into skilled labor used for technology adoption and skilled labor used for R&D. Following Anzoategui et al. (2017), this allocation is endogenously determined, by the adoption gap $z_{it} - a_{it}$

$$l_{sit} = (z_{it} - a_{it}) l_{sait} + l_{srit} \quad \forall\, i. \tag{28}$$

A representative household again maximizes lifetime utility w.r.t. consumption and labor supply, given by

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - \frac{1}{\mu_{ut}\varsigma_u} \frac{l_{ut}^{1+\varphi}}{1+\varphi} - \frac{1}{\mu_{st}\varsigma_s} \frac{l_{st}^{1+\varphi}}{1+\varphi} \right] \quad \forall\, i, \tag{29}$$

where β is the intertemporal discount factor, σ is the relative risk aversion coefficient, and φ is the marginal disutility w.r.t. labor supply. Assuming that households own the firms, they maximize this utility function subject to the intertemporal budget constraint. Following Comin (2009), with μ_{ut} and μ_{st} preference shifter shocks are introduced to shock the labor supply. These shocks can also be interpreted as capturing frictions in the labor market and taxes. The shocks follow stationary stochastic processes

$$\log \mu_{ut} = \rho_{\mu_u} \log \mu_{u,t-1} + \epsilon_t^{\mu_u}, \tag{30}$$

$$\log \mu_{st} = \rho_{\mu_s} \log \mu_{s,t-1} + \epsilon_t^{\mu_s}. \tag{31}$$

Equation	Definition
$l_t = l_{ut} + l_{st}$	labor Aggregation (Optional)
$l_{ut}^{\varphi} = \varsigma_u \mu_u \frac{w_{ut}}{c_t^{\sigma} p_t}$	Unskilled Labor Supply
$l_{st}^{\varphi} = \varsigma_s \mu_s \frac{w_{st}}{c_t^{\sigma} p_t}$	Skilled Labor Supply
$c_t^{-\sigma} = \beta E_t[c_{t+1}^{-\sigma}(1 - \delta + r_{t+1})]$	Euler Equation
$c_{it} = c_t \omega_i \left(\frac{p_{it}}{p_t}\right)^{-\epsilon} \quad \forall i$	Optimal Consumption Choice
$l_{uit} = l_{ut} \varsigma_{ui} \left(\frac{w_{uit}}{w_{ut}}\right)^{\nu_u} \quad \forall i$	Optimal Unskilled labor Allocation
$l_{sit} = l_{st} \varsigma_{si} \left(\frac{w_{sit}}{w_{st}}\right)^{\nu_s} \quad \forall i$	Optimal Skilled labor Allocation
$w_{ut} = \left[\sum_{i=1}^{N} \varsigma_{ui} w_{uit}^{1-\nu_u}\right]^{\frac{1}{1-\nu_u}}$	Average Unskilled Wage Rate
$w_{st} = \left[\sum_{i=1}^{N} \varsigma_{si} w_{sit}^{1-\nu_s}\right]^{\frac{1}{1-\nu_s}}$	Average Skilled Wage Rate
$k_{t+1} = (1 - \delta)k_t + i_t$	Capital Law of Motion
$y_{it} = a_{it}^{\frac{1}{\psi_i - 1}} \theta_{it} k_{it}^{\alpha_i} l_{uit}^{\beta_i} M_{it}^{1-\alpha_i-\beta_i} \quad \forall i$	Production Function Sector i
$M_{it} = \left[\sum_{j=1}^{N} \gamma_{ji}^{\frac{1}{\eta_i}} m_{jit}^{\frac{\eta_i - 1}{\eta_i}}\right]^{\frac{\eta_i}{\eta_i - 1}} \quad \forall i$	Intermediate Inputs Sector i
$k_{it} = a_{it}^{\frac{1}{1-\psi_i}} \alpha_i y_{it} \frac{MC_{it}}{r_t p_t} \quad \forall i$	Demand for Capital Sector i
$l_{uit} = a_{it}^{\frac{1}{1-\psi_i}} \beta_i y_{it} \frac{MC_{it}}{w_{uit}} \quad \forall i$	Demand for Labor Sector i
$m_{jit} = a_{it}^{\frac{\eta_i}{1-\psi_i}} (1 - \alpha_i - \beta_i)^{\eta_i} y_{it}^{\eta_i} \left(\frac{MC_{it}}{p_{jt}}\right)^{\eta_i} \gamma_{ji} M_{it}^{1-\eta_i} \quad \forall i \; \forall j$	Demand for sector j, Sector i
$p_t = \left[\sum_{i=1}^{N} \omega_i p_{it}^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$	Ideal Price Index
$p_{M_{it}} = \left[\sum_{j=1}^{N} \gamma_{ji} p_{jt}^{1-\eta_i}\right]^{\frac{1}{1-\eta_i}} \quad \forall i$	Price of Intermediates Sector i

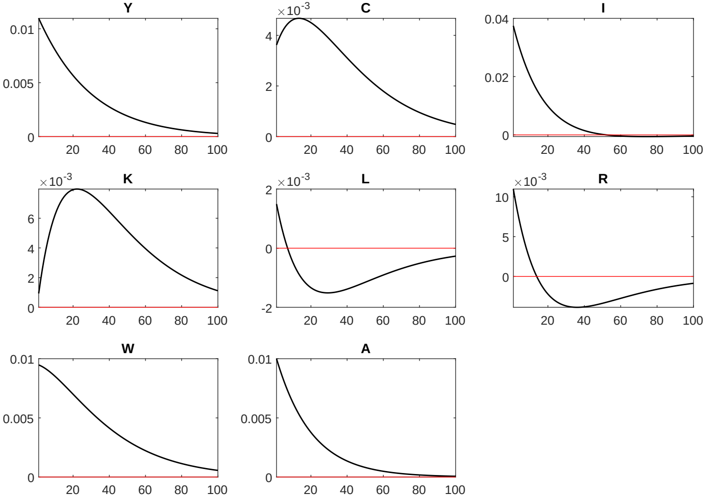
$MC_{it} = \frac{1}{\theta_{it}} \left(\frac{r_t p_t}{\alpha_i}\right)^{\alpha_i} \left(\frac{w_{uit}}{\beta_i}\right)^{\beta_i} \left(\frac{p_{M_{it}}}{1-\alpha_i-\beta_i}\right)^{1-\alpha_i-\beta_i} \quad \forall i$	Marginal Cost Sector i
$p_{it} = a_{it}^{\frac{1}{1-\psi_i}} \frac{\psi_i}{\psi_i - 1} MC_{it} \quad \forall i$	(Optimal) Price Level Sector i
$\lambda_{it} = \kappa_i \left(\omega_{adi} \sum_{j=1}^N \gamma_{ji} a_{jt} + \omega_{aui} \sum_{j=1}^N \gamma_{ij} a_{jt}\right)^{\rho_{Mai}} (z_{it} l_{sait})^{\rho_{ai}} \quad \forall i$	Adoption Success Probability Sector i
$\Pi_{it} = p_{it} y_{it} - w_{uit} l_{uit} - r_t p_t k_{it} - \sum_{j=1}^N p_{jt} m_{jit} \quad \forall i$	Intermediate Goods Aggregate Profit Sector i
$v_{it}^a = \Pi_{it} + \phi_i E_t \frac{v_{i,t+1}^a a_{it}}{a_{i,t+1}(1+r_{t+1})} \quad \forall i$	Value of Adopted Technology Sector i
$J_{it}^z = E_t \left\{ \frac{\lambda_{it} v_{i,t+1}^a \frac{z_{it}}{a_{i,t+1}} + (1-\lambda_{it}) J_{i,t+1}^z \frac{z_{it}}{z_{i,t+1}}}{1+r_{t+1}} \right\} - w_{sit} l_{sait} z_{it} \quad \forall i$	Value of Unadopted Technology Sector i
$w_{sit} l_{sait} = \rho_{ai} \lambda_{it} \phi_i E_t \left\{ \frac{\frac{v_{i,t+1}^a}{a_{i,t+1}} - \frac{J_{i,t+1}^z}{z_{i,t+1}}}{1+r_{t+1}} \right\} \quad \forall i$	Optimal Adoption Investment Sector i
$a_{i,t+1} = \lambda_{it} \phi_i [z_{it} - a_{it}] + \phi_i a_{it} \quad \forall i$	Evolution of Adopted Technology Sector i
$\vartheta_{it} = \chi_{it} z_{it} \left(\omega_{rdi} \sum_{j \neq i} \gamma_{ji} z_{jt} + \omega_{rui} \sum_{j \neq i} \gamma_{ij} z_{jt}\right)^{\rho_{Mri}} l_{srit}^{\rho_{zi}-1} \quad \forall i$	Productivity of R&D sector i
$E_t \frac{\frac{\vartheta_{it}}{z_{i,t+1}} J_{i,t+1}^z}{1+r_{t+1}} = w_{sit} \quad \forall i$	Optimal R&D Investment Sector i
$z_{i,t+1} = \phi_i z_{it} + \vartheta_{it} l_{srit} \quad \forall i$	Evolution of Technology Sector i
$l_{sit} = (z_{it} - a_{it}) l_{sait} + l_{srit} \quad \forall i$	Skilled labor Aggregation Sector i
$y_{it} = c_{it} + i_{it} + \sum_{j=1}^N m_{ijt} \quad \forall i$	Equilibrium Condition Sector i
$\log \chi_{it} = (1 - \rho_{\chi_i}) \log \chi_i^* + \rho_{\chi_i} \log \chi_{i,t-1} + \epsilon_{it}^{\chi} \quad \forall i$	R&D Shock Sector i
$\log \theta_{it} = \rho_{\theta_i} \log \theta_{i,t-1} + \epsilon_{it}^{\theta} + \epsilon_t \quad \forall i$	Productivity Shock Sector i
$\log \mu_{ut} = \rho_{\mu_u} \log \mu_{u,t-1} + \epsilon_t^{\mu_u}$	Unskilled labor Supply Shock
$\log \mu_{st} = \rho_{\mu_s} \log \mu_{s,t-1} + \epsilon_t^{\mu_s}$	Skilled labor Supply Shock
$k_t = \sum_{i=1}^N k_{it}$	Capital Aggregation
$i_t = \sum_{i=1}^N i_{it}$	Investment Aggregation
$y_t = \sum_{i=1}^N y_{it}$	Output Aggregation (Optional)

Simulation: A Textbook RBC Model

The model is defined by 8 equations in 8 endogenous variables (y, c, k, l, i, w, r, a):

Equation	Definition
$c_t^{\sigma} l_t^{\varphi} = w_t$	Labor Supply
$c_t^{-\sigma} = \beta E_t[c_{t+1}^{-\sigma}(1 - \delta + r_{t+1})]$	Euler Equation
$k_{t+1} = (1 - \delta)k_t + i_t$	Capital Law of Motion
$y_t = a_t k_t^{\alpha} l_t^{1-\alpha}$	Production Function
$k_t = \alpha y_t / r_t$	Demand for Capital
$l_t = (1 - \alpha) y_t / w_t$	Demand for Labor
$y_t = c_t + i_t$	Equilibrium Condition
$\log a_t = (1 - \rho)a^* + \rho \log a_{t-1} + \epsilon_t$	Technology Shock

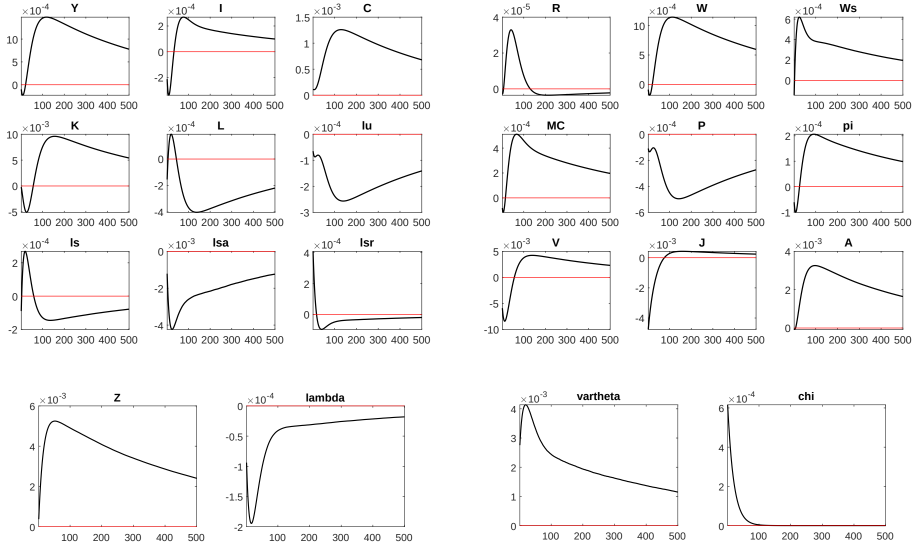
Impulse Response Functions Following 0.1 sd Productivity Shock (a_t):



Simulation: RBC Model with Endogenous Technology

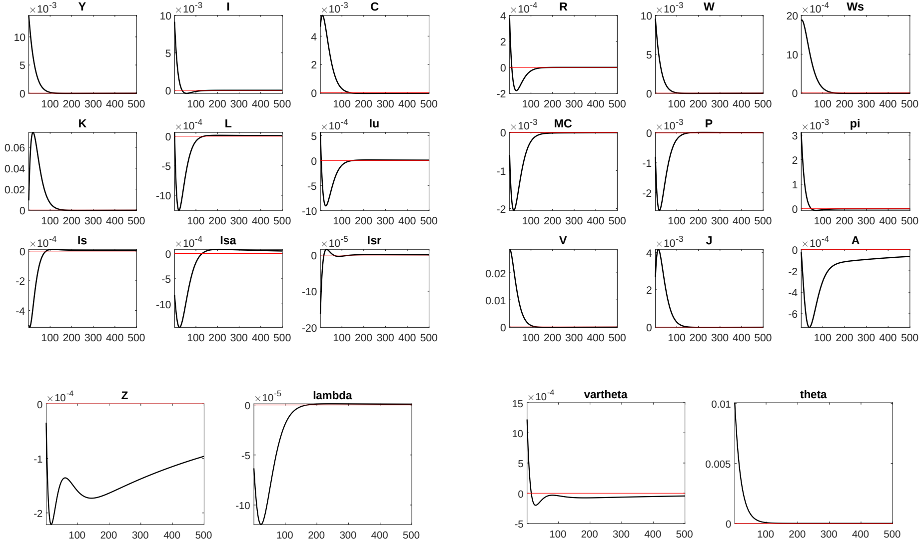
Impulse Response Functions Following 0.1 sd R&D Shock (χ)

Using a 1st-order Taylor Expansion of the model (calibrated to the US economy following Anzoategui et al. (2017)) around the steady-state, with stochastic simulation over 2000 periods (200 periods burn-in).



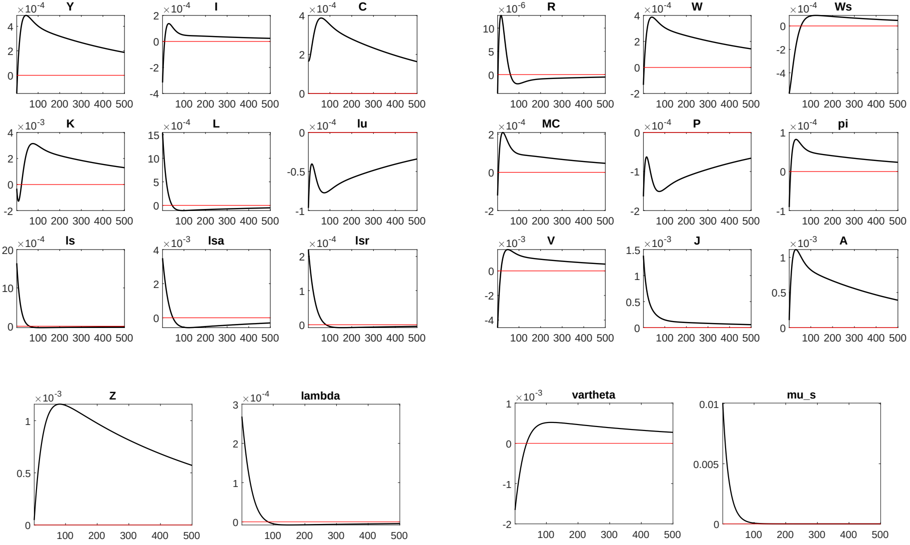
Impulse Response Functions Following 0.1 sd Productivity Shock (θ)

Using a 1st-order Taylor Expansion of the model (calibrated to the US economy following Anzoategui et al. (2017)) around the steady-state, with stochastic simulation over 2000 periods (200 periods burn-in).



Impulse Response Functions Following 0.1 sd Skilled Labor Supply Shock (μ_s)

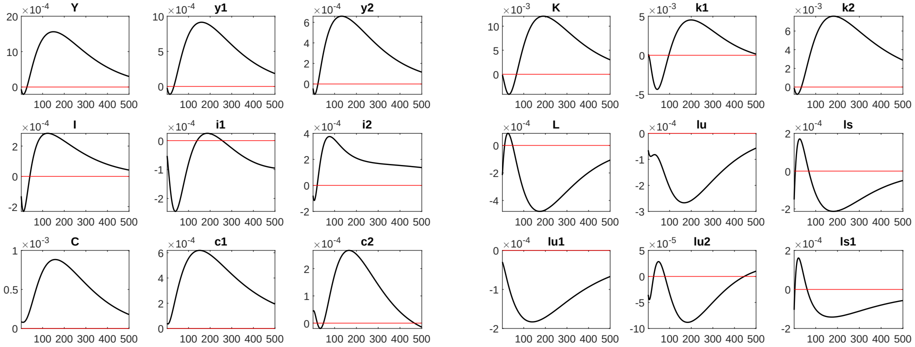
Using a 1st-order Taylor Expansion of the model (calibrated to the US economy following Anzoategui et al. (2017)) around the steady-state, with stochastic simulation over 2000 periods (200 periods burn-in).

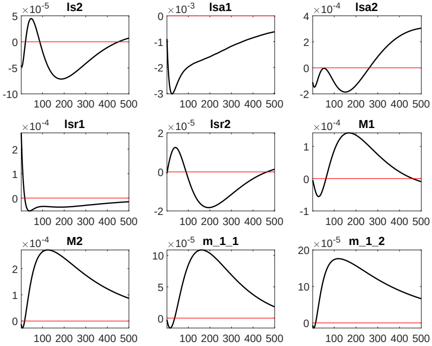


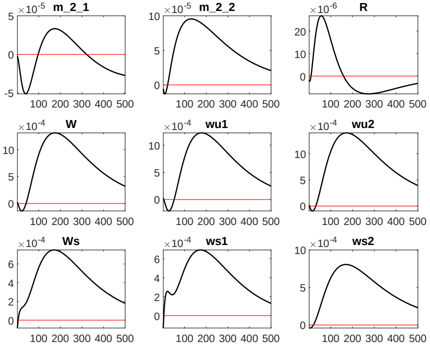
Simulation: 2-Sector RBC with Endogenous Technology

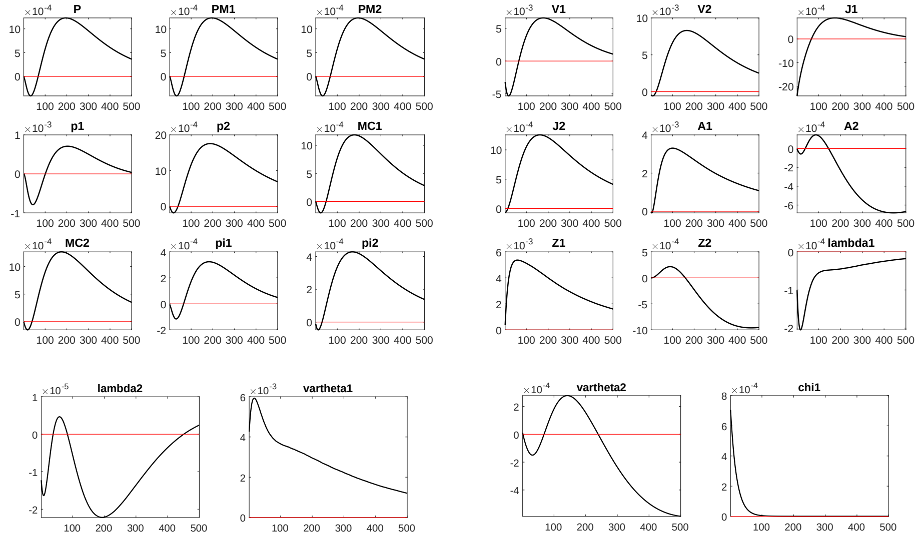
IRF's Following 0.1 sd R&D Shock to Sector 1 (χ_1) - No Spillovers

Using a 1st-order Taylor Expansion of the model (symmetric stylized calibration) around the steady-state, with stochastic simulation over 20,000 periods (200 periods burn-in).



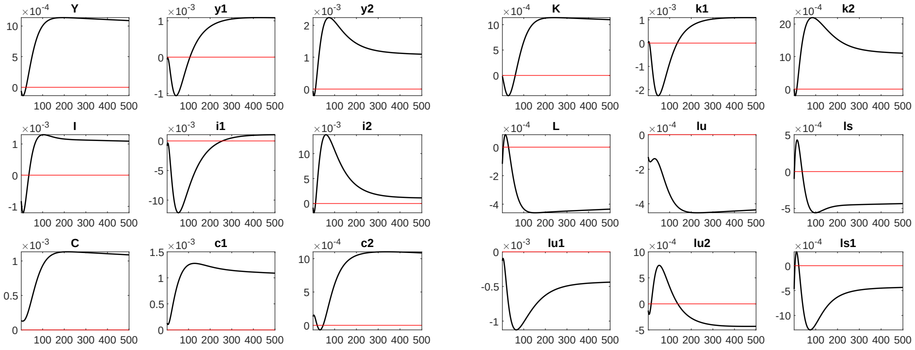

Is2
Isa1
Isa2
Isr1
Isr2
M1
M2
m_1_1
m_1_2

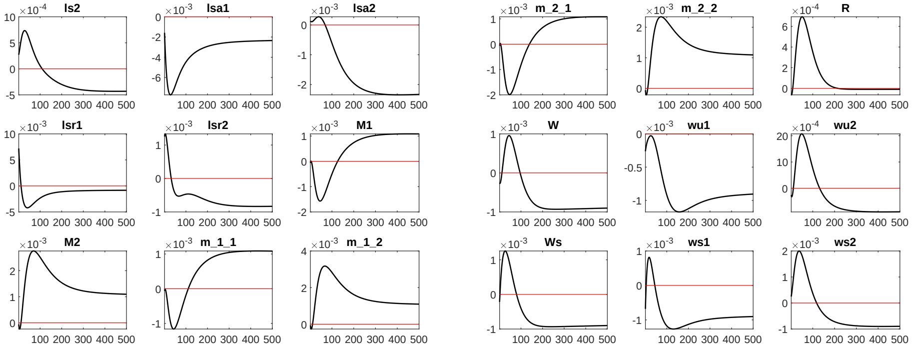

m_2_1
m_2_2
R
W
wu1
wu2
Ws
ws1
ws2

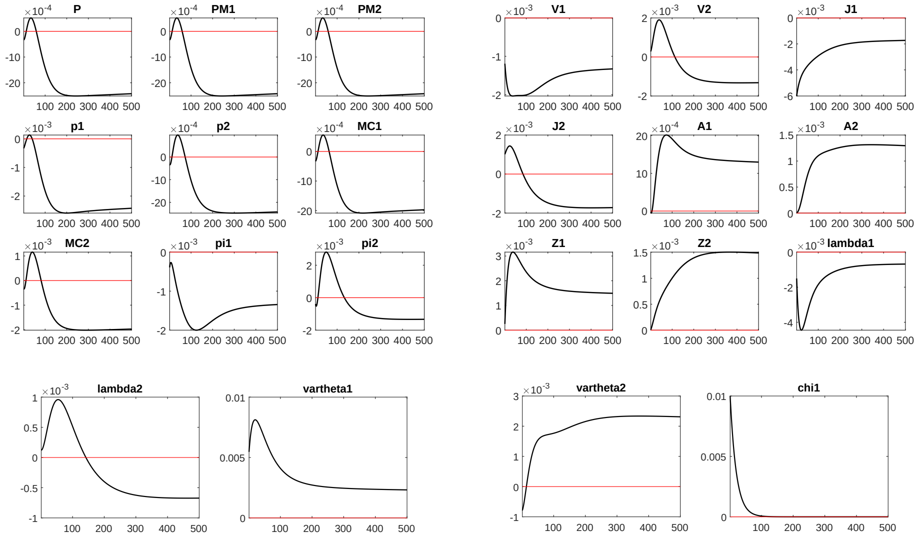

×10-4 P
×10-4 PM1
×10-4 PM2
×10-3 V1
×10-3 V2
×10-4 J1
×10-3 p1
×10-4 p2
×10-4 MC1
×10-4 J2
×10-3 A1
×10-4 A2
×10-4 MC2
×10-4 pi1
×10-4 pi2
×10-3 Z1
×10-4 Z2
×10-4 lambda1
×10-5 lambda2
×10-3 vartheta1
×10-4 vartheta2
×10-4 chi1

IRF's Following 0.1 sd R&D Shock to Sector 1 (χ_1) - With R&D Spillovers ($\rho_{Mri} = 0.2 \ \forall \ i$), and Adoption Spillovers ($\rho_{Mai} = 0.1 \ \forall \ i$)

Using a 1st-order Taylor Expansion of the model (symmetric stylized calibration) around the steady-state, with stochastic simulation over 20,000 periods (200 periods burn-in).




ls2
lsa1
lsa2
m_2_1
m_2_2
R
lsr1
lsr2
M1
W
wu1
wu2
M2
m_1_1
m_1_2
Ws
ws1
ws2
100 200 300 400 500


P
PM1
PM2
V1
V2
J1
p1
p2
MC1
J2
A1
A2
MC2
pi1
pi2
Z1
Z2
lambda1
lambda2
vartheta1
vartheta2
chi1

Conclusion

- The model generates IRF's $> 5x$ more persistent than conventional RBC IRF's in response to R&D and skilled labor supply shocks, approx. resembling the length of the Medium-Term Cycle.

- All sectors in a productive network benefit in the long-term from R&D shocks to one sector. If there are spillovers, closely linked sectors benefit even more than the sector doing R&D. Overall there are quite interesting and complex interactions as technology diffuses through the multi-sector economy.

- R&D and skilled labor supply shocks to a single sector have prolonged aggregate economic effects via these complimentarities.

Further Research

- Does the introduction of NK frictions such as price- and wage-stickiness or investment adjustment costs change the observed sectoral responses and the distribution of gains from the R&D shock in some critical respect?

- Need to attempt calibration of the model to a real input-output network a la Horvath (2000) and Atalay (2017) and see what it can describe in terms of real short-and medium-run fluctuations:

  - For some of the complex unobserved technological parameters, bayesian estimation will be necessary, with (preferably quarterly) series of VA, labor and R&D spending for each sector $\rightarrow$ ambitious requirements for disaggregated data.

  - Probably necessary to add some bells and whistles and perhaps change equations a bit to really give a good fit of the data similar to the model of Anzoategui et al. (2017) $\rightarrow$ long way to go to obtain a good DSGE model.

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